THE

MATHEMATICAL GAZETTE.

EDITED BY

W. J. GREENSTREET, M.A.

WITH THE CO-OPERATION OF

F. S. MACAULAY, M.A., D.Sc., AND PROF. E. T. WHITTAKER, M.A., F.R.S.

LONDON:

G. BELL AND SONS, LTD., PORTUGAL STREET, KINGSWAY, AND BOMBAY.

VOL. IX. MARCH, 1917. No. 128.

The Mathematical Association.

THE Annual Meeting of the Mathematical Association was held on Friday, 5th January, 1917, at the London Day Training College, Southampton Row, London, W.C., the President, Professor A. N. Whitehead, Sc.D., F.R.S., in the chair.

(1) The following Report of the Council for the year 1916 was adopted:

REPORT OF THE COUNCIL FOR 1916.

During the year 1916, 25 new members have been elected, and the number of members now on the Roll is 672. Of these 9 are honorary members, 36 are life members by composition, 39 are life members under the old rule, and 588 are ordinary members. The number of associates is about 170.

The Council regret to have to record the deaths of *Second-Lieutenant the Rev. A. G. T. Alderson, Mr. S. Andrade, the *Rev. G. S. Arnold Wallinger (Lance-Corporal, Inns of Court O.T.C.), *Captain G. P. Blake, Mr. M. S. David, *Corporal G. Dawson, R.E., *Captain C. F. Ellerton, Mr. B. E. Hammond, *Second-Lieutenant F. L. Henley, Mr. C. S. Jackson, Mr. H. T. Kelsey, Mr. J. B. Parish, Mr. Charles Smith (Master of Sidney Sussex College, Cambridge), the Right Honourable Sir James Stirling, F.R.S., *Captain J. K. Tully, *Mr. S. A. White, Dr. B. Williamson, F.R.S., and Mr. J. Stanton Wise. Those marked with an asterisk were serving with His Majesty's Forces; Mr. Parish was one of the original members of the Association when it was founded in 1871; Mr. Hammond joined the Association a year later in 1872, and Dr. Williamson in 1873.

Mr. Jackson had for many years been a member of the Council, and retired a year ago in accordance with the rules. It was intended that he should be proposed for re-election in 1917. He was also the first chairman of the London Branch, and did much to ensure its success. The Association has benefited greatly by his keen interest in, and his clear grasp of, everything which could contribute to the cause of mathematical training. Many members of the Association will doubtless desire to endorse the expression of deep sympathy which the President sent to Mrs. Jackson and her family on behalf of the Council.

Sir George Greenhill has very generously presented to the Library a complete set of the volumes of the *Bulletin of the American Mathematical Society*. The Council tender to the donor the Association's very grateful thanks for his useful and valuable gift.

The Council again recommend that the regulations which govern the elections of the Teaching Committees be temporarily suspended, and that the term of office of the existing Committees be extended for one year.

The Council have had under consideration the case of those members who are serving with His Majesty's Forces, and have decided to recommend that they be allowed to retain their membership without the payment of any subscription while so serving, and that the *Gazette* be sent to them as usual if they so wish.

The report of the Girls' Schools Committee, on which that Committee has been engaged for some time, has now been completed, and will shortly be issued to the members.

Professor A. N. Whitehead, Sc.D., F.R.S., retires at this meeting from the office of President. The Council, in the name of the Association, desire to record their deep sense of the valuable services which he has rendered to the Association during the past two years, and to thank him very cordially for his close personal attention to its interests. As Professor Whitehead's successor the Council have the pleasure of nominating Professor T. P. Nunn, M.A., D.Sc., Vice-Principal of the London Day Training College, to be President for the years 1917 and 1918. They also nominate Professor Whitehead to be a Vice-President of the Association.

Miss E. Greene and Dr. F. S. Macaulay retire now from the Council, and are not eligible for re-election for the coming year. The members present at the Annual Meeting will be asked to nominate and elect others to fill the two vacancies.

Mr. R. F. Davis, who has audited the accounts of the Association for the last twenty years, has intimated his wish to retire from that office, and the Council feel that the Association would like to join with them in expressing their thanks to Mr. Davis

and their very warm appreciation of his long and valued service. Dr. Macaulay has kindly consented to audit the accounts for the year 1916.

The Council again desire to acknowledge the indebtedness of the Association to Mr. Greenstreet for his services as Editor of the *Mathematical Gazette*: and to offer their thanks to the authorities of the London Day Training College for their kindness in affording accommodation for the Annual Meeting, and for the meetings of the Council and of the Committees which have been held during the year.

---

(2) The Treasurer's Report for the year 1916 was read and adopted.

(3) The Teaching Committees.

The term for which the Teaching Committees were elected in 1914 expired in 1916, and had already been extended for one year. It was proposed and agreed that the regulations which govern the elections be temporarily suspended and that the term of office of the Committees be extended for another year.

(4) The Election of Officers and Council for the year 1917.

Professor T. P. Nunn, M.A., D.Sc., was nominated and elected as President for the years 1917 and 1918.

The retiring President (Prof. Whitehead) was elected a Vice-President.

Miss J. M. Lewis, of Wycombe Abbey School, and Dr. W. P. Milne, of Clifton College, were nominated and elected on the Council in place of Miss E. Greene and Dr. F. S. Macaulay.

(5) An address on "The School Syllabus in Geometry," by Professor T. P. Nunn.

(6) A Short Account of "Some of the Work of the Teaching Committee," by Mr. A. W. Siddons.

The Presidential Address was delivered at the opening of the afternoon meeting.

(7) "Technical Education and its relation to Literature and Science," by Professor A. N. Whitehead.

(8) "An Accuracy Test set in some Public Schools," by Mr. A. W. Siddons.

(9) "The Place of Mathematics in Educational Reconstruction," by Mr. P. Abbott.

For the purpose of continuing the discussion on this subject short papers on special aspects of it were read:

(*a*) "Mathematics after the War," by Rev. E. M. Radford.

(*b*) "Should we continue to teach Geometry?" by Mr. C. J. L. Wagstaff.

(*c*) "Mathematics in Secondary Schools," by Mr. W. J. Dobbs.

PRESIDENTIAL ADDRESS.

By A. N. Whitehead, F.R.S.

TECHNICAL EDUCATION AND ITS RELATION TO SCIENCE AND LITERATURE.

The subject of this address is Technical Education. I wish to examine its essential nature and also its relation to a liberal education. Such an enquiry may help us to realise the conditions for the successful working of a national system of technical training. It is also a very burning question among mathematical teachers; for mathematics is included in most technical courses.

Now it is unpractical to plunge into such a discussion without forming in our minds the best ideal result towards which we desire to work, however modestly we may frame our hopes as to the result which in the near future is likely to be achieved.

People are shy of formulated ideals; and accordingly we find formulation of the ideal state of mankind placed by a modern dramatist * in the mouth of a mad priest. "In my dreams it is a country where the State is the Church and the Church the people: three in one and one in three. It is a commonwealth in which work is play and play is life: three in one and one in three. It is a temple in which the priest is the worshipper and the worshipper the worshipped: three in one and one in three. It is a godhead in which all life is human and all humanity divine: three in one and one in three. It is, in short, the dream of a madman."

Now the part of this speech to which I would direct attention is embodied in the phrase, 'It is a commonwealth in which work is play, and play is life.' This is the ideal of technical education.

It sounds very mystical when we confront it with the actual facts, the toiling millions, tired, discontented, mentally indifferent, and then the employers—I am not undertaking a social analysis, but I shall carry you with me when I admit that the present facts of society are a long way off this ideal. Furthermore, we are agreed that an employer who conducted his workshop on the principle that 'work should be play,' would be ruined in a week.

The curse that has been laid on humanity, in fable and in fact, is that by the sweat of its brow shall it live. But reason and moral intuition have seen in this curse the foundation for advance. The early Benedictine monks rejoiced in their labours, because they were thereby made fellow-workers with Christ. Stripped of its theological trappings, the essential idea remains, that work should be transfused by intellectual and moral vision, and thereby turned into a joy, triumphing over its weariness and its pain.

* Bernard Shaw, cf. *John Bull's Other Island.*

Each of us will restate this abstract formulation in a more concrete shape in accordance with his private outlook. State it how you like, so long as you do not lose the main point in your details. However you phrase it, it remains the sole real hope of toiling humanity; and it is in the hands of technical teachers and of those who control their spheres of activity, so to mould the nation that daily it may pass to its labours in the spirit of the monks of old.

The immediate need of the nation is a large supply of skilled efficient workmen, of men with inventive genius, and of employers alert in the development of new ideas. Another essential condition is industrial peace.

There is only one way to obtain these admirable results. It is by producing workmen, men of science, and employers who enjoy their work. View the matter practically, in the light of our knowledge of average human nature. Is it likely that a tired, bored workman, however skilful his hands, will produce a large output of first-class work? He will limit his production and be an adept at evading inspection; he will be slow in adapting himself to new methods; he will be a focus of discontent, full of impractical revolutionary ideas, controlled by no sympathetic apprehension of the real working of trade conditions. If, in the troubled times which may be before us, you wish appreciably to increase the chance of some savage upheaval, introduce widespread technical education and ignore the Benedictine ideal. Society will then get what it deserves. Again, inventive genius requires pleasurable mental activity as a condition for its vigorous exercise. 'Necessity is the mother of invention' is a silly proverb. 'Necessity is the mother of futile dodges' is much nearer to the truth. The basis of the growth of modern invention is science, and science is almost wholly the outgrowth of pleasurable intellectual curiosity.

The third class are the employers, who are to be enterprising. Now it should be observed that it is the successful employers who are the important people to get at, the men with business connections all over the world, men who are already rich. No doubt there will always be a continuous process of rise and fall of businesses. But it is futile to expect flourishing trade, if in the mass the successful businesses are suffering from atrophy. Now if the successful men conceive their businesses as merely indifferent means for acquiring other disconnected opportunities of life, they have no spur to alertness. They are already doing very well, the mere momentum of their business engagements will carry them on for their time. They are not at all likely to bother themselves with the doubtful chances of new methods. Their real soul is in the other side of their life. Desire for money will produce hardfistedness and no enterprise. There is much more hope for humanity from manufacturers who enjoy their work than from those who continue in irksome business with the object of founding hospitals.

Finally, there can be no prospect of industrial peace so long as masters and men in the mass conceive themselves as engaged in a soul-less operation of extracting money from the public. Enlarged views of the work performed and of communal service thereby rendered can be the only basis on which to found sympathetic co-operation.

The conclusion to be drawn from this discussion is that, alike for masters and for men, a technical or technological education which is to have any chance of satisfying the practical needs of the nation must be conceived in a liberal spirit as a real intellectual enlightenment as to principles applied and the services rendered. In such an education geometry and poetry are as essential as turning-lathes.

The mythical figure of Plato may stand for modern liberal education as does that of St. Benedict for technical education. We need not entangle ourselves in the qualifications necessary for a balanced representation of the actual thoughts of the actual men. They are used here as symbolic figures typical of antithetical notions. We consider Plato in the light of the type of culture he now inspires. In its essence a liberal education is an education for thought and for æsthetic appreciation. It proceeds by imparting a knowledge of the masterpieces of thought, of imaginative literature, and of art. The action which it contemplates is command. It is an aristocratic education, implying leisure. This Platonic ideal has rendered imperishable services to European civilisation. It has encouraged art, it has fostered that spirit of disinterested curiosity which is the origin of science, it has maintained the dignity of mind in the face of material force, a dignity which claims freedom of thought. Plato did not, like St. Benedict, bother himself to be a fellow worker with his slaves; but he must rank with Benedict among the emancipators of mankind. His type of culture is the peculiar inspiration of the liberal aristocrat, the class from which Europe derives what ordered liberty it now possesses. For centuries, from Pope Nicholas V. to the schools of the Jesuits, and from the Jesuits to the modern headmasters of English schools, this educational ideal has had the strenuous support of the clergy.

For certain people it is a very good education. It suits their type of mind and the circumstances amid which their life is passed. But more has been claimed for it than this. It has been represented as the ideal education, and every curriculum has been judged adequate or defective according to its approximation to this sole type.

The essence of the type is a large discursive knowledge of the best literature. The ideal product of the type is the man who is acquainted with the best that has been written. Such a man will have acquired the chief languages, he will have considered the histories of the rise and fall of nations, the varied poetic expression of human feeling, and have read the great dramas and novels.

He will also be well grounded in the chief philosophies, and have attentively read those philosophic authors who are distinguished for lucidity of style.

It is obvious that, except at the close of a long life, he will not have much time for anything else, if any approximation is to be made to the fulfilment of this programme. One is reminded of the calculation in a dialogue of Lucian that before a man could be justified in practising any one of the current ethical systems, he should have spent 150 years in examining their credentials.

Such ideals are not for human beings. What is meant by a liberal culture is nothing so ambitious as a full acquaintance with the varied literary expression of civilized mankind from Asia to Europe, and from Europe to America. A small selection only is required, but then, as we are told, it is a selection of the very best. I have my doubts of a selection which includes Xenophon and omits Confucius, but then I have read neither in the original.

The ambitious programme of a liberal education really shrinks to a study of some fragments of literature, included in a couple of important languages.

But the expression of the human spirit is not confined to literature. There are the other arts and there are the sciences. Also education must pass beyond the passive reception of the ideas of others. Powers of initiative must be strengthened. Unfortunately, initiative does not mean just one acquirement. There is initiative in thought, initiative in action, and the imaginative initiative of art; and these three categories require many subdivisions.

The field for acquirement is large, and the individual so fleeting and so fragmentary. Classical scholars, scientists, headmasters, are all equally ignoramuses.

There is a curious illusion that a more conplete culture was possible when there was less to know. Surely the only gain was that it was more possible to remain unconscious of ignorance. It cannot have been a gain to Plato to have read neither Shakespeare, nor Newton, nor Darwin. The achievements of a liberal education have in recent times not been worsened. The change is, that its pretensions have been found out.

My point is that no course of study can claim any position of ideal completeness. Nor are the omitted factors of subordinate importance. The insistence in the Platonic culture on disinterested intellectual appreciation is a psychological error. Action, and the transition of events amid the inevitable bond of cause to effect, are fundamental. An education which strives to divorce intellectual or æsthetic life from these fundamental facts carries with it the decadence of civilisation. Essentially, culture should be for creative action, and its effect should be to divest labour of the associations of aimless toil. Art exists in order that we may know the deliverances of our senses as good. It heightens the sense-world.

Again, disinterested scientific curiosity is a passion for an ordered intellectual vision of the connections of events. But the goal of such curiosity is the marriage of action to thought. This essential intervention of action even in abstract science is often overlooked. No man of science wants merely to know. He acquires knowledge to appease his passion for discovery. He does not discover in order to know, he knows in order to discover. The pleasure which art and science can give to toil is the pleasure which arises from successfully directed intention. Also it is this same pleasure which is yielded to the scientist and to the artist.

The antithesis between a technical and a liberal education is fallacious. There can be no adequate technical education which is not liberal, and no liberal education which is not technical, that is, no education which does not impart both technique and intellectual vision. In simpler language, education should turn out the pupil with some things he knows well, and some things he can do well. This intimate union of practice and theory aids both. The intellect does not work best in a vacuum; the stimulation of creative impulse requires, especially in the case of a child, the quick natural transition to practice. Geometry and mechanics, followed by workshop practice, gain that reality without which mathematics is verbiage.

There are three main methods which are required in a national system of education, the literary curriculum, the scientific curriculum, the technical curriculum.

But each one of these curricula should include the other two. What I mean is that every form of education should give the pupil a technique, a science, an assortment of general ideas, an æsthetic appreciation, and that each of these sides of his training should be illuminated by the others. Lack of time, even for the most favourable pupil, makes it impossible to develop fully each curriculum. Always there must be a dominant emphasis. The most direct æsthetic training naturally falls in the technical curriculum, in those cases when the technique is that requisite for some art or artistic craft. But it is of high importance in both a literary and a scientific education.

The educational method of the literary curriculum is the study of language, that is the study of our most habitual method of conveying to others our states of mind. The technique which should be acquired is the technique of verbal expression; the science is the study of the structure of language and the analysis of the relations of language to the states of mind conveyed. Furthermore, the subtle relations of language to feeling, and the high development of the sense organs to which spoken and written words appeal, lead to keen æsthetic appreciations being aroused by the successful employment of language.

Finally the wisdom of the world is preserved in the masterpieces of linguistic composition.

This curriculum has the merit of homogeneity. All its various parts are coordinated and play into each others' hands. We can hardly be surprised that such a curriculum, when once broadly established, should have claimed the position of the sole perfect type of education.

Its defect is unduly to emphasise the importance of language. Indeed, the varied importance of verbal expression is so overwhelming, that its sober estimation is difficult. Recent generations have been witnessing the retreat of literature and of literary forms of expression from their position of unique importance in intellectual life. In order truly to become a servant and a minister of nature something more is required than literary aptitudes.

A scientific education is primarily a training in the art of observing natural phenomena and in the knowledge and deduction of laws concerning the sequence of such phenomena. But here, as in the case of a liberal education, we are met by the limitations imposed by shortness of time. There are many types of natural phenomena, and to each type there corresponds a science with its peculiar modes of observation and with its peculiar types of thought employed in the deduction of laws. A study of science in general is impossible in education, all that can be achieved is the study of two or three allied sciences. Hence the charge of narrow specialism urged against any education which is primarily scientific. It is obvious that the charge is apt to be well-founded; and it is worth considering how, within the limits of a scientific education, and to the advantage of such an education, the danger can be avoided.

Such a discussion requires the consideration of technical education. A technical education is in the main a training in the art of utilising knowledge for the manufacture of material products. Such a training emphasises manual skill, and the coordinated action of hand and eye, and judgment in the control of the process of construction. But judgment necessitates knowledge of those natural processes of which the manufacture is the utilisation. Thus somewhere in technical training, an education in scientific knowledge is required. If you minimise the scientific side, you will confine it to the scientific experts, if you maximise it you will also impart it in some measure to the men, and—what is of no less importance—to the directors and managers of businesses.

Technical education is not necessarily allied exclusively to science on its mental side. It may be an education for an artist or for apprentices to an artistic craft. In that case æsthetic appreciation will have to be cultivated in connection with it.

An evil side of the platonic culture has been its total neglect of technical education as an ingredient in the complete development of ideal human beings. This neglect has arisen from two disastrous antitheses, namely that between mind and body, and that between thought and action. I will here interject, solely to avoid criticism,

that I am well aware that the Greeks highly valued physical beauty and physical activity.

I lay it down as an educational axiom that in teaching you will come to grief as soon as you forget that your pupils have bodies. This is exactly the mistake of the post-renaissance platonic curriculum. But nature can be kept at bay by no pitchfork; so in English education, being expelled from the class-room, she returned with a cap and bells in the form of all-conquering athleticism.

The connections between intellectual activity and the body, though diffused in every bodily feeling, are focussed in the eyes, the ears, the voice, and the hands. There is a coordination of senses and thought, and also a reciprocal reaction between brain activity and material creative activity. In this reaction the hands are peculiarly important. It is a moot point whether the human hand created the human brain, or the brain created the hand. Certainly the connection is intimate and reciprocal. Such deep-seated relations are not widely atrophied by a few hundred years of disuse in exceptional families.

The disuse of hand-craft is a contributory cause to the brain-lethargy of aristocracies, which is only mitigated by sport where the concurrent brain activity is reduced to a minimum, and the hand craft lacks subtlety. The necessity for constant writing and vocal exposition is some slight stimulus to the thought-power of the professional classes. Great readers, who exclude other activities, are not distinguished by subtlety of brain. They tend to be timid conventional thinkers. No doubt this is partly due to their excessive knowledge outrunning their powers of thought; but partly it is due to the lack of brain-stimulus from the productive activities of hand or voice.

In estimating the importance of technical education we must rise above the exclusive association of learning with book learning. First-hand knowledge is the ultimate basis of intellectual life. To a large extent book learning conveys second-hand information, and as such can never rise to the importance of immediate practice. Our goal is to see the immediate events of our lives as instances of our general ideas. What the learned world tends to offer is one second-hand scrap of information illustrating an idea derived from another second-hand scrap of information. The second-handedness of the learned world is the secret of its mediocrity. It is tame because it has never been scared by facts. The main importance of Francis Bacon's influence does not lie in any peculiar theory of inductive reasoning which he happened to express, but in the revolt against second-hand information of which he was a leader.

The peculiar merit of a scientific education should be that it bases thought upon first-hand observation; and the corresponding merit of a technical education is that it follows our deep natural

instinct to translate thought into manual skill, and manual activity into thought.

We are a mathematical association, and it is natural to ask 'Where do we come in ?' We come in just at this point.

The thought which science evokes is logical thought. Now logic is of two kinds, the logic of discovery and the logic of the discovered.

The logic of discovery consists in the weighing of probabilities, in discarding details deemed to be irrelevant, in divining the general rules according to which events occur, and in testing hypotheses by devising suitable experiments. This is inductive logic.

The logic of the discovered is the deduction of the special events which under certain circumstances would happen in obedience to the assumed laws of nature. Thus when the laws are discovered or assumed, their utilisation entirely depends on deductive logic. Without deductive logic science would be entirely useless. It is merely a barren game to ascend from the particular to the general, unless afterwards we can reverse the process and descend from the general to the particular, ascending and descending like the angels on Jacob's ladder. When Newton had divined the law of gravitation he at once proceeded to calculate the earth's attractions on an apple at its surface and on the moon. We may note in passing that inductive logic would be impossible without deductive logic.

Now mathematics is nothing else than the more complicated parts of the art of deductive reasoning, especially where it concerns number, quantity, and space. In the teaching of science, the art of thought should be taught: namely, the art of forming clear conceptions applying to first-hand experience, the art of divining the general truths which apply, the art of testing divinations, and the art of utilising general truths by reasoning to more particular cases of some peculiar importance. Furthermore, a power of scientific exposition is necessary so that the relevant issues from a confused mass of ideas can be stated clearly, with due emphasis on important points.

By the time a science, or small group of sciences, has been taught thus amply, with due regard to the general art of thought, we have gone a long way towards correcting the specialism of science. The worst of a scientific education based, as is necessarily the case, on one or two particular branches of science, is that the teachers under the influence of the examination system are apt merely to stuff their pupils with the narrow results of those special sciences. It is essential that the generality of the method be continually brought to light and contrasted with the speciality of the particular application. A man who only knows his own science, as a routine peculiar to that science, does not even know that. He has no fertility of thought, no power of quickly seizing the bearing of alien

ideas. He will discover nothing and will be stupid in every practical application.

This exhibition of the general in the particular is extremely difficult to effect, especially in the case of younger pupils. The art of education is never easy. To surmount its difficulties, especially those of elementary education, is a task worthy of the highest genius. It is the training of human souls.

Mathematics, well taught, should be the most powerful instrument in gradually implanting this generality of idea. The essence of mathematics is perpetually to be discarding more special in favour of more general ideas, and special methods in favour of general methods. We express the conditions of a special problem in the form of an equation, but that equation will serve for a hundred other problems, scattered through diverse sciences. The general reasoning is always the powerful reasoning, because deductive cogency is the property of abstract form. There again we must be careful. We shall ruin mathematical education if we use it merely to impress general truths. The general ideas are the means of connecting particular results. After all it is the concrete special cases which are important. Thus in the handling of mathematics, in your results you cannot be too concrete, and in your methods you cannot be too general. The essential course of reasoning is to generalise what is particular and then to particularise what is general. Without generality there is no reasoning, without concreteness there is no importance.

Concreteness is the strength of technical education. I would remind you that truths which lack the highest generality are not necessarily concrete facts. For example, $x+y=y+x$ is an algebraic truth more general than $2+2=4$. But 'two and two make four' is itself a highly general proposition lacking any element of concreteness. To obtain a concrete proposition immediate intuition of a truth concerning particular objects is requisite, for example 'these two apples and those two apples together make four apples' is a concrete proposition, if you have direct perception or immediate memory of the apples.

In order to obtain the full realisation of truths as applying, and not as empty formulae, there is no alternative to technical education. Mere passive observation is not sufficient. In creation only is there vivid insight into the properties of the object produced. If you want to understand anything, make it yourself, is a sound rule. Your faculties will be alive, your thoughts gain vividness by an immediate translation into acts. Your ideas gain that reality which comes from seeing the limits of their application.

In elementary education this doctrine has long been put into practice. Young children are taught to familiarise themselves with shapes and colours by simple manual operations of cutting out and of sorting. But good though this is, it is not quite what I mean.

That is practical experience before you think, experience antecedent to thought in order to create the ideas, a very excellent discipline. But technical education should be much more than that; it is creative experience while you think, experience which realises your thought, experience which teaches you to coordinate act and thought, experience leading you to associate thought with foresight and foresight with achievement. Technical education gives theory, and a shrewd insight as to where theory fails.

A technical education is not to be conceived as a maimed alternative to the perfect platonic culture, namely, as a defective training unfortunately made necessary by cramped conditions of life. No human being can attain to anything but fragmentary knowledge, and a fragmentary training of his capacities. There are, however, three main roads along which we can proceed with good hope of advancing towards the best balance of intellect and character, these are the way of literary culture, the way of scientific culture, the way of technical culture. No one of these methods can be exclusively followed without grave loss of intellectual activity and of character. But a mere mechanical mixture of the three curricula will produce bad results in the shape of scraps of information, never interconnected or utilised. We have already noted as one of the strong points of the traditional literary culture that all its parts are coordinated. The problem of education is to retain the dominant emphasis, whether literary, scientific, or technical, and without loss of coordination to infuse into each way of education something of the other two.

To make definite the problem of technical education fix attention on two ages, one thirteen when elementary education ends and the other seventeen when technical education ends, so far as it is comprised in a school training. These dates give four years for a technical course. I am aware that for artisans in junior technical schools a three years' course would be more usual. On the other hand, for naval officers, and for the directing classes generally a longer time can be afforded. We want to consider the principles to govern a curriculum which shall land these children at the age of seventeen in the position of having technical skill useful to the community.

Their technical manual training should start at thirteen bearing a modest proportion to the rest of their work, and should increase in each year, finally to attain to a substantial proportion. Above all things it should not be too specialised. Workshop finish and workshop dodges adapted to one particular job should be taught in the commercial workshop, and should form no essential part of the school course. A properly trained worker would pick them up in no time. In all education the main cause of failure is staleness. Technical education is doomed if we conceive it as a system for catching children young and for giving them one highly specialised

manual aptitude. The nation has need of a fluidity of labour, not merely from place to place, but also, within the reasonable limits of allied aptitudes, from one special type of work to another special type. I know that here I am on delicate ground, and I am not claiming that men while they are specialising on one sort of work should spasmodically be set to other kinds. That is a question of trade organisation with which educationalists have no concern. I am only asserting the principles that the training should be broader than the ultimate specialisation, and that the resulting power of adaptation to varying demands is advantageous to the workers, to the employers, and to the nation.

In considering the intellectual side of the curriculum we must be guided by the principle of the coordination of studies. In general, the intellectual studies most immediately related to the manual training will be some branches of science. More than one branch will, in fact, be concerned, and even if that be not the case, it is impossible to narrow down scientific study to a single thin line of thought. It is possible, however, provided that we do not press the classification too far, roughly to classify technical pursuits according to the dominant science involved. We thus find a sixfold division, namely,

(1) Geometrical techniques; (2) Mechanical techniques; (3) Physical techniques; (4) Chemical techniques; (5) Biological techniques; (6) Techniques of Commerce and of Social Service.

By this division it is meant that, apart from auxiliary sciences, some particular science requires emphasis in the training for most occupations. We can, for example, reckon carpentry, ironmongery, and many artistic crafts among geometrical techniques. Similarly, agriculture is a biological technique. Probably cookery, if it includes food catering, would fall midway between biological, physical, and chemical sciences, though of this I am not sure.

The sciences associated with commerce and social service would be partly algebra, including arithmetic and statistics, and partly geography and history. But their section is somewhat heterogeneous in its scientific affinities. Anyhow, the exact way in which technical pursuits are classified in relation to science is a detail. The essential point is that with some thought it is possible to find scientific courses which illuminate most occupations. Furthermore, the problem is well understood, and has been brilliantly solved in many of the schools of technology and junior technical schools throughout the country.

In passing from science to literature in our review of the intellectual elements of technical education, we note that many studies hover between the two; for example, history and geography. They are both of them very essential in education, provided that they are the right history and right geography. Also books giving descriptive accounts of the general results and trains of thought in various

sciences fall in the same category. Such books should be partly historical and partly expository of the main ideas which have finally arisen. Prof. R. A. Gregory's recent book, *Discovery*, and the Home University Library series illustrate my meaning. Their value in education depends on their quality as mental stimulants. They must not be inflated with gas on the wonders of science, and must be informed with a broad outlook.

It is unfortunate that the literary element in education has rarely been considered apart from grammatical study. The historical reason is that, when the modern platonic curriculum was being formed, Latin and Greek were the sole keys which rendered great literature accessible. But there is no necessary connection between literature and grammar. The great age of Greek literature was already past before the arrival of the grammarians of Alexandria. Of all types of men to-day existing, classical scholars are the most remote from the Greeks of the Periclean times.

Mere literary knowledge is of slight importance. The only thing that matters is how it is known. The facts related are as nothing. Literature only exists to express and develop that imaginative world which is our life, the kingdom which is within us.

It follows that the literary side of a technical education should consist in an effort to make the pupils enjoy literature. It does not matter what they know, but the enjoyment is vital. The great English Universities, under whose direct authority school children are examined in plays of Shakespeare, to the certain destruction of their enjoyment, should be prosecuted for soul-murder.

Now there are two kinds of mental enjoyment, the enjoyment of creation and the enjoyment of relaxation. They are not necessarily separated. A change of occupation may give the full tide of happiness which comes from the concurrence of both forms of pleasure.

The appreciation of literature is really creation. The written word, its music, and its associations are only the stimuli. The vision which they evoke is our own doing. No one, no genius other than our own, can make our own life live. But except for those engaged in literary occupations, literature is also a relaxation. It gives exercise to that other side which any occupation must suppress during working hours. It also has the same function in life as has literature.

To obtain the pleasure of relaxation requires no help. The pleasure is merely to cease doing. Some such pure relaxation is a necessary condition of health. Its dangers are notorious, and to the greater part of the necessary pure relaxation nature has affixed not enjoyment, but the oblivion of sleep.

Creative enjoyment is the outcome of successful effort and requires help for its initiation. Such enjoyment is necessary for high-speed work and for original achievement. To speed up production with unrefreshed workmen is a disastrous economic policy. Temporary

success will be at the expense of the nation, which for long years of their lives will have to support worn-out artisans, unemployables. Equally disastrous is the alternation of spasms of effort with periods of pure relaxation. Such periods are the seed-time of degeneration, unless rigorously curtailed. The normal recreation should be change of activity satisfying the cravings of other instincts. Games afford such activity. Their disconnection emphasises the relaxation, but their excess leaves us empty.

It is here that literature and popular art should play an essential part in a healthily organised nation. Their services to economic production would be only second to those of sleep or of food. I am not now talking of the training of an artist but of the use of art as a condition of healthy life. It is analogous to sunshine in the physical world.

When we have once rid our minds of the idea that knowledge is to be exacted, there is no special difficulty or expense involved in helping the growth of artistic enjoyment. All school-children could be sent at regular intervals to neighbouring theatres where suitable plays could be subsidised. Similarly for concerts and kinema films. Pictures are more doubtful in their popular attraction. But interesting representations of scenes or ideas which the children have read about would probably appeal. The pupils themselves should be encouraged in artistic efforts. Above all the art of reading aloud should be cultivated. The Roger de Coverley essays of Addison are perfect examples of readable prose.

Art and literature have not merely an indirect effect on the main energies of life. Directly, they give vision. The world spreads wide beyond the deliverances of material sense, with subtleties of reaction and with pulses of emotion. Vision is the necessary antecedent to control and to direction. In the contest of races, which in its final issues will be decided in the workshops and not on the battle-field, the victory will belong to those who are masters of stores of trained nervous energy, working under conditions favourable to growth. One such essential condition is Art.

If there had been time, there are other things which I should like to have said ; for example, to advocate the inclusion of one foreign language in all education. From direct observation I know this to be possible for artisan children. But enough has been put before you to make plain the principles with which we should undertake national education.

In conclusion, I recur to the thought of the Benedictines who saved for mankind the vanishing civilisation of the ancient world, by linking together knowledge, labour, and moral energy. Our danger is to conceive practical affairs as the kingdom of evil, in which success is only possible by the extrusion of ideal aims. I believe that such a conception is a fallacy, directly negatived by experience. In education this error takes the form of a mean

view of technical training. Our forefathers in the dark ages saved themselves by embodying high ideals in great organisations. It is our task, without servile imitation, boldly to exercise our creative energies, remembering amid discouragements that the coldest hour immediately precedes the dawn.

In response to a call from the President, Mr. P. Abbott gave his paper on:

THE POSITION OF MATHEMATICS IN EDUCATIONAL RECONSTRUCTION.

It would not be possible in the short time allotted to me for this paper to discuss any scheme of Educational Reconstruction which might appear to me as ideal. Indeed it is doubtful, notwithstanding the constant use which is made of the term, whether any reconstruction of our educational system, in the full sense of the word, is probable or even possible. But many reforms, some of them fundamental and far-reaching in their effects, are both possible and probable. It is with certain of these which, directly or indirectly, will materially affect the position of mathematics, that I propose to deal as far my limited time allows.

Before that is done, it is essential that we should first examine, however briefly, the fundamental causes which are tending to produce change. They are, in the main, though not entirely, the outcome of the war, which has compelled us to scrutinise our systems and methods, and to test their values from the standpoint of efficiency. Also, in view of the increasing strain upon our national resources and the depletion of our reserves of wealth, we are compelled to eliminate all forms of waste both now and after the war.

Examining our educational system from these aspects, we cannot feel satisfied that all is well with it, and hence there is a very general feeling that it must be made more efficient. The country has been impressed with the efficiency of Germany, not only from the military point of view, but also in the matters of education, science and industry. With this has also come the realisation that peace will bring with it yet another struggle, in industry and commerce, not less fierce nor less intense than the present clash of arms. For that we must make due preparation.

Moreover, the war has led the average person to realise to what an extent science, and the applications of science to engineering, electrical engineering, chemistry and the like, have permeated our modern existence. There is evidence everywhere of an increasing desire to acquire some clear knowledge of these things, and especially that the youth of the nation shall be trained to understand them. A feeling for more scientific and technical education is growing, and is finding expression, not only by the usual signs in the press, but in the number of students, surprisingly large under the circumstances, who are attending our technical schools. Let our lady members note that this movement is not confined to the male sex, for women students are beginning to make their appearance in our classes in engineering, electrical engineering, architecture and industrial chemistry. There has begun a shifting of the centre of gravity in education towards science and technology, and this cannot fail to influence the teaching of mathematics.

These are some of the currents and movements which are making for changes, and we must be prepared to take account of them if mathematics is to occupy its proper position in our educational system. We must not assume that changes which may be the outcome of these movements will mean any deterioration in our standards or any lower-

ing of ideals. We can adapt our mathematical teaching to scientific and technical requirements, we can mould it so that its applications and examples are closely related to our civic life and to the phenomena of our environment, and we shall profit by the change.

Specific Reforms.

Of the specific reforms which will result, or may result, from the causes to which I have alluded, I propose to deal only with four as being those which will mostly directly influence the teaching of mathematics. These are:

(1) Compulsory attendance at day continuation classes of all boys and girls to the age of seventeen or eighteen.
(2) Re-adjustment of curricula.
(3) Reforms in examinations.
(4) Developments in the training of teachers.

Compulsory Continuation Classes.

The most important of these and the most far-reaching in its consequences is the first, viz. compulsory attendance at day continuation classes. I am not concerned for the moment with the extremely difficult questions of all kinds which such a reform will raise; nor can I stop to point out how it will effect secondary, technical and university education. I will merely assume that it is a reform which will probably be accomplished, and which we must therefore be prepared to consider in so far as it will affect the teaching of mathematics.

The fundamental fact for our consideration seems to me to be this—that, for the first time, there will come under the control of the state the continuous education of every boy and girl between infancy and the age of seventeen or eighteen. This opens up great and attractive possibilities in regard to the expansion and continuous development of the various subjects of the school curriculum. Consider, for example, the case of mathematics under the existing atrophied system of state education, in which the instruction of the majority of the nation ceases abruptly at the age of thirteen or fourteen; it is usually impossible to attempt anything more than ordinary arithmetic. But with the accomplishment of the reform which I have indicated, it will be possible for practically every boy and girl in the country to receive a continuous and methodically developed course in the fundamentals of mathematics. I would, therefore, like to place this proposition before you.

That under a system of compulsory continued education to the age of seventeen or eighteen, a minimum course in mathematics should in general form an essential part of the curriculum of every boy and girl.

Such a claim cannot be put forward unless it can be justified on grounds which are sound from an educational point of view, and convincing to the public. There are other subjects in the curriculum for which expansion will be sought, and the case for mathematics must be a strong one if it is to be treated as is suggested. The claim can only be maintained if the course of mathematics, which is proposed, conforms in the main to the movements and requirements which I have previously indicated, and at the same time is educationally sound. It seems to me, therefore, that any such course must be based upon the following fundamental principles:

Fundamental Principles of a Mathematics Course.

(1) It should provide the mathematics required for an elementary scientific and technical training.

(2) It should include a suitable amount of training in abstract reasoning.

(3) It should largely be based upon experimental work in the early stages, and to a less extent in the later stages, and should include a training in accurate measurement and the use of measuring instruments.

(4) It should provide such mathematical knowledge as is necessary for the intelligent discharge of the duties of a citizen.

(5) It should at every stage be closely associated with the environment of the child and such phenomena of life as are within his comprehension.

The essential characteristics of the course should be, that it should be thorough and practical. We should not attempt to cover too much ground, but what is done should be so thorough that mathematics becomes to the pupil a tool which he can use with confidence and certainty.

A course which will satisfy the conditions which I have laid down can only be arrived at after careful and prolonged consideration; but, as it is necessary to provide material for discussion, I will proceed with considerable hesitancy to outline such a scheme.

A Minimum Course in Mathematics.

Arithmetic and Mensuration. A sound and practical training in computation, use of tables, calculation of cost, and the use of decimal and vulgar fractions.

Ratio, proportion and proportional quantities.

Square root.

Averages and percentages.

Commercial applications such as interest and investments.

Applications such as are necessary for a proper understanding of the national and local government, *e.g.* rates and taxes, statistics, the National Debt, etc.

Mensuration of plane figures such as quadrilaterals, triangles, and circle.

Mensuration of solids such as prisms, pyramids, cone and sphere.

Measurements. Training in accurate measurements, including weighing. The use of measuring instruments such as slide callipers, micrometer screw gauges, vernier, etc.

Algebra. The meaning and use of formulae. Such algebraical laws and methods of manipulation as are necessary for the use of formulae.

Easy equations.

Meaning and use of the sine, cosine, and tangent of an angle. Use of tables.

Solution of right-angled triangles.

Graphical representation of statistics and simple functions. Meaning of a graph and its gradient.

Determination of the law or equation of a curve in simple cases.

Coordinates as a means of determining position.

The use of logarithms (without a formal treatment of the laws of indices).

Geometry. Study of a small compact body of the more important truths which occur in elementary geometry.

Notes. 1. Much of the arithmetical work would be founded upon experiments performed by the child. Mensuration would provide a number of such experiments, but the work should not be confined to small manufacturers' models, but should extend to objects which occur in everyday life.

2. Similarly, mensuration and the phenomena of a child's life may be used to provide examples of formulae, so that, in the first instance, the child learns to manipulate formulae which he has himself constructed, and which have some concrete meaning to him.

3. Solids would be extensively used throughout, especially in geometry, the study of which ought to begin with solids.

Secondary Schools.

Such is the kind of minimum course in mathematics which is suggested as being generally suitable for those who will pass through the primary and continuation schools, *i.e.* the bulk of the nation. But in the case of secondary schools it has always been possible, of course, to secure a continuous course of mathematics extending over a period of years. The only question which we have to consider is —what will be the effect on the course of the new movements and conditions to which I have alluded ? The full consideration of this is not possible within the limits of this paper. But there are a few general observations which I may be permitted to make.

In the first place, one of the effects of the introduction of compulsory continuation schools will probably be an increase in the average duration of the school life in the secondary schools. Thus the total amount of time available for mathematics will be increased. In the second place, it is just as necessary that the secondary schools should take account of those movements and tendencies of which I have spoken, as the primary and continuation schools, and the mathematics must be moulded accordingly. A considerable amount of the existing teaching of mathematics in our secondary schools is in need of radical improvement. It is out of touch with modern developments, it is lacking in vitality, and it is disappointing in its results. It fails on the side of utility, while as a form of mental training it is a sham. Reforms must, I think, follow the lines indicated above. On examining the principles previously enumerated as governing a minimum "course in mathematics," I cannot find anything which can be disregarded when the secondary school course is considered. Nor, if the minimum course itself be examined, will it be found that there is anything which can be omitted in the case of secondary schools. The only material difference would be that a greater amount of mathematics would be attempted in the secondary school, and, consequently, additions would have to be made to the minimum course, additions which would have in view the needs of those who would proceed to higher work at the university, or higher technical institution, or the like. It should also include a course in mechanics, treated, in the main, experimentally.

Moreover, it is most undesirable that there should be any material difference in the nature of the mathematics taught in the two systems. There will be difference in method of treatment, in the distribution of the emphasis, and in the omission or inclusion of certain topics. But it is imperative, I submit, that in any scheme of reconstruction the general treatment of the subject should be on similar lines. Education will not end in all cases even with the continuation school or the secondary school. In technical education, at least, the products of both will go on together to higher work, and hence true coordination is essential or there will be discordance, waste, and loss of efficiency.

It may reasonably be objected that the teaching of mathematics along the lines which I have indicated is scarcely possible under the existing conditions of examinations, staff and equipment. But we are assuming that there is going to be some reconstruction of our educational system, and hence I venture to mention, briefly, certain changes

which I believe to be necessary if the teaching of mathematics is to be placed upon a satisfactory basis.

Examinations.

In the first place our examinations need further reforms. On that point I need say nothing to this Association, in view of the strenuous efforts it has made to secure reforms in this direction, happily with a considerable measure of success. All the signs of the times go to show that important reforms are probable in the near future which will materially affect all school examinations, and there is good reason for hoping that they will not be quite such a brake upon progress as they have been in the past.

Mathematical Laboratories.

The practical work in mathematics and mechanics which is advocated in this paper will necessitate a certain amount of apparatus and equipment. The only really effective way of dealing with this is to equip special mathematical laboratories. The equipment need not be expensive, nor the apparatus elaborate. Much of it is better made on the spot, and for many purposes models constructed by the pupils themselves are much to be preferred to the manufacturer's elaborate, varnished, standardised productions. If a specially fitted laboratory cannot be obtained, it might be possible to set aside a room to be used specially for practical work. Failing that, much can be accomplished in the classroom, but I would like to urge strongly that, in considering any scheme of reconstruction in mathematical teaching, a mathematical laboratory should be regarded as essential.

Training of Teachers.

My last proposal deals with the teacher himself, and with his training as a teacher. It is, I venture to think, a grave defect in our educational system that there has been an almost complete lack of training in the case of teachers entering the secondary and technical branches of the profession. There has been practically no provision whatever for the training of technical teachers, while among secondary teachers a comparatively small number of women and a still smaller number of men have been trained. It is a curious circumstance that while most would agree that teaching is one of the most highly skilled of the professions, yet so little is done to ensure the proper acquisition of that skill. In this connection it cannot be too widely known that after 1920 nobody will be accepted for registration by the Teachers' Registration Council unless he or she has undergone an approved course of training. To decry, as I have heard some do, the necessity for a pedagogical training is to make the admission that the profession is not a highly skilled one, a fatal obstacle to securing proper recognition and status for any profession. To carry the matter further, it is not only necessary, in my judgment, that a teacher should be professionally trained, but if he is to be a "specialist" teacher, and most are specialists in these days, he should receive a "specialised" training. For example, a man or a woman who intends to be a teacher of mathematics should not only receive a general pedagogical training, but should also be *trained to teach mathematics*. This would also, I think, mean that his knowledge of mathematics should be broader and more general than that of the man who intends to become a mathematician pure and simple. For example, he might make some study of statistics or public finance ; he should also have received some training in some branch of science or technology in which the application of mathematics

figure prominently. Conversely, the teacher of physics or engineering should go through a special course in mathematics. In this way only will we secure effective correlation of mathematical and scientific teaching, an ideal which is so easily and glibly spoken of, but which, in practice, so seldom exists. Perhaps this last of my points is the most important of all. Take care of your teacher and education will take care of itself.

The President called upon Mr. C. J. L. WAGSTAFF for his paper on :

SHOULD WE CONTINUE TO TEACH GEOMETRY ?

GEOMETRY in a Mathematical Syllabus seems to me to occupy much the same position that Greek does in the Classical. The two have similar merits and similar defects, and I am doubtful whether either deserves its place except in specialist classes.

With no ill-will to the Classics, many believe that classical teaching absorbs too much time. Some assert that while few boys acquire anything of real value from it, many are made dull and listless, and, after suffering from a diet of stones when their desire was for bread, come to regard all school work as distasteful and valueless : eventually, without exactly knowing why, they will come to despise their school education as their parents do.

It is not my business now to discuss Greek, but as those who live in glass-houses should not throw stones, I think members of the Mathematical Association, and especially those who are in any way connected with education, should look round carefully to see if their own dwellings are stone proof. For myself, I must acknowledge, after much sorrowful reflection, that almost everything that "reformers" say, of Greek, I am compelled to say of geometry.

Both are excellent : intellectual delights, humanising influences ; mental gymnastics, and so on *ad nauseam*. But only to the very few does either appeal at all. Neither is taught with any useful end in view. The ordinary boy learns nothing from geometry and regards it as pompous rubbish. Why *should* he "prove" that two sides of a triangle are greater than the third, that the opposite sides of a parallelogram are equal—except to please his master, or rise above the middle of his class ? He knows most of the facts quite well and regards them as obvious.

We tell ourselves that geometry is excellent training in reasoning and logic, but other branches of mathematics give the same training in much more instructive and useful form ; and further, geometrical reasoning is of such a particular kind and based on such restricted premises as to be useless in our daily life. We have no special confidence in the judgment of a geometrician in questions of practical politics.

Perhaps I shall prevent misconception if I indicate what I mean by geometry. I do not mean mensuration and practical measurement, nor do I mean the study of geometrical conics. But there is a ladder which is used to connect the two, and it is that which I have in mind. I believe it wrong to put boys to climb it who we know will fail to reach the top. Let the few boys we think may climb it have their chance, but don't spoil the ninety-nine for the sake of the hundredth.

The ninety-nine—if for them we abolish the ladder—will still have their useful knowledge of geometry. They won't walk home by curly paths from ignorance of the properties of a straight line, and they will still measure the diagonals of their picture frames to test their

right angles: they will know Pythagoras and neglect Ptolemy, but they will be freed from much needless toil.

There is a popular belief that Euclid was dead and buried twenty years ago. He certainly never died, and if buried was immediately exhumed and reincarnated. Book shops do not sell Euclid now as such, they stock dozens of "Geometries"; nearly all of them copies of dear old Todhunter: fatter and better dressed perhaps, but still essentially the same. Euclid knew no sines or cosines, + and −, × and ÷. Roots and indices were absent from his pages. His body-snatchers have been nearly as conservative and pure.

I have been looking lately at several school "Geometries." I have little opportunity of actually listening to class lessons in geometry, and am therefore obliged to form opinions from listening to discussions, studying modern text-books, and criticising examination papers. Except for the boy—the hundredth of whom I have spoken—who has a special geometrical kink in his brain, how dull they are! How far removed from events of daily life, and what a waste of time! They take many long pages over saying that the area of a triangle equals half the product of base and altitude; so many in fact that even if they do get there, the boys do not realise they are only writing long what they writ short in their "Practical Measurement" lessons.

In one of these "Geometries," which seems in no way different from its many brethren, out of the first fifty-seven theorems which are supposed to be proved, eighteen are so obvious that proofs can only be useless lumber except for a philosopher, eight are repeats, two are not proofs in any sense of the word, seven are wholly needless, three I cannot classify, and only nineteen can, in my opinion, be seriously set before a class as suitable for mental digestion. And after all these fifty-seven theorems, what has been realised? Pythagoras, area of a triangle—known practically to all boys—appear to be the only things of value. Proportion, rectangles contained by segments of a chord, sections, solids may be found in other volumes, but, as they have yet to be presented to me by the publishers, I am unable to give you any analysis of them.

Let us stop teaching boys (and most certainly girls) these fifty-seven theorems. For the real geometrician they can be condensed from the 200 pages occupied at present to less than twenty, and so studied when convenient.

The taint of geometry has spread to other subjects. We have tried—generally with complete success—to reduce mechanics and trigonometry to its level, and to deal with them by means of propositions, text-books, ink and paper. That saves us labour, and the teacher of mathematics is now as completely furnished within the four walls of his class-room as his classical colleague. Frictionless wheels and trigonometrical identities, triangles of force, weightless ropes have all the merits of Greek irregular verbs to the complete schoolmaster: and all their faults to the schoolboy.

If these subjects are to be taught, let us teach them with theodolites and prismatic compasses, with real ropes, pulleys and engines. Let the boys enjoy their lessons in mathematics just as much as the hours they spend in the workshops, and with a result as stimulating and beneficial.

To sum up, I think geometry can no longer claim a prominent place in our scheme of mathematics. In the old days when classics and mathematics were the only serious subjects to be considered the teaching of it was justifiable, but it does not now suit modern requirements. It takes up time urgently needed for other parts of mathematics: it leads nowhere: it cannot plead special consideration as

mental gymnastic apparatus: to most boys it is dull, and success in it means no more than success in solving acrostics.

In its place I would plead for much more practical subjects: practical mechanics, for instance; we might teach a workshop trigonometry to boys of twelve, and let every boy of fourteen use his logs and slide rule. The effort to understand the use of these would give all the old intellectual training, and they would be useful and lifelong possessions. Can we say the same for Greek geometry?

There is, as far as I can see, only one way in which we can be free from the toils of geometry. Treat it as most of us would treat Greek, and let it be optional or non-existent in all—even in "mathematical-pass" examinations.

After ten years of freedom we might have grown a school system of mathematics—we have now a system of mathematical cupboards—and be in a position to see if it is desirable to bind ourselves again with our present chains.

Mr. W. J. Dobbs then read his paper on:

MATHEMATICS IN SECONDARY SCHOOLS.

Aim. I take it that the object of teaching mathematics in a boys' school is to develop, in combination, the power (i) to apply mathematics successfully to matters of human interest, (ii) to appreciate in some measure that "realm of order" (the expression is borrowed from Professor Nunn) in which mathematical ideas exist.

Method. For school purposes the subject mathematics is best developed as a tool applied to practical use, the gradual evolution of the instrument taking place as the scope of its action extends. In this way neither the weaker boys, nor those possessing special mathematical ability, are sacrificed. By hanging on slavishly, blindly, to an effete tradition, we alienate the sympathies of the vast majority of boys, we subject even those who possess a natural aptitude for mathematics to serious risk of blindness to the real significance of their work, and we allow most boys to leave school before they have learned to put their mathematics to practical use.

Curriculum. By selecting those topics in which the practical value of mathematics is most apparent, the skilful teacher can find the opportunities required to direct the attention of his pupils to those theoretical considerations which are of paramount importance, while, at the same time, he answers the natural question of the schoolboy—"what is it for?" The principle is as old as the hills, and was enunciated, with the loftiest application, by a very well-known writer many centuries ago in these words—"*τὰ γὰρ ἀόρατα ἀπὸ κτίσεως κόσμου τοῖς ποιήμασι νοούμενα καθορᾶται*" ("For, ever since the world was created, the unseen things are clearly seen, being perceived through the things that are made.")

It may be desirable to postpone specialisation in mathematics until after the time of the general school examination (the suggested First Examination of the Board of Education)—say about the age of sixteen. If so, a school course of mathematics should be taken as one subject, not the thing of shreds and patches hitherto insisted upon. This involves the scrapping of many topics beloved of the external examiner, or, at any rate, their postponement to a later stage.

Stage 1. Age 10 to 12. In this stage the mathematical work will be mainly *Arithmetic*. Regular *daily* practice in figuring is indispensable. If a boy does not acquire by the age of twelve the power to

work quickly and accurately easy questions in straightforward arithmetic, he will be seriously handicapped throughout his school career. But it is desirable that some part of the work in *Arithmetic* should be used to introduce the elements of *Algebra* and *Geometry*. The algebra and geometry should grow out of the arithmetic,—*Algebra* as generalised arithmetic, *Geometry* in connection with measurement, simple surveying, drawing, folding, etc.—any formal treatment being only gradually introduced and adapted to the capacity of the pupils. If the manual instruction and elementary science are in the hands of other masters, there should be close conscious co-operation between the different masters, the data for many mathematical problems being taken from, or suggested by, the practical work.

The formal treatment which is gradually developed in the later stages should be on lines which run parallel to the less formal treatment adopted in the early stage. This involves close co-operation between the masters who take the juniors and the master who takes the senior boys.

Stage 2. Age 12 to 14. It is desirable that some part of the science teaching should be in the hands of the mathematical master, and the subject which lends itself best to this treatment is *Mechanics*—a most valuable subject for boys, and most valuable also to the teacher in developing mathematics as an instrument for humanistic study. But generally in this stage the mechanics will be treated experimentally mainly by the physics master. If so, this should be done in close co-operation with the mathematical master, so that the experimental treatment may run on lines for the most part parallel to the more formal treatment adopted later.

In this stage more and more of the *Arithmetic* will give place to *Algebra, Geometry, Mensuration* and *Mechanics*, including the use of at least one *Trigonometrical* function. It is undesirable that six trigonometrical functions should be introduced at once. Some teachers prefer to begin with the *Tangent*, but personally I prefer the *Cosine* as the most useful and valuable. If the geography master co-operated, and he should do so, he will probably prefer the *Cosine*.

But it is not the formal elaboration of these as separate subjects which is required. *Algebra, Geometry, Trigonometry*, etc., should not emerge as separate subjects until the *specialist* stage after the time of the general school examination. The work should continue to be largely numerical, and it is pedantic and most undesirable to keep the idea of measurement out of the geometry or to forbid the use of algebraic symbols. The custom of external examinations in setting separate papers on algebra and geometry is a great obstacle to progress on these lines.

If the science teaching during this stage involves a course of *Weighing* and *Measuring* and *Experimental Mechanics*, taken by the physics master, the data for many mathematical problems should be supplied by him. In this way considerable saving of time may be effected.

Stage 3. Age 14 to 16. In this stage the *Arithmetic* may be still further curtailed, and the *Trigonometry* considerably developed. The boys should now learn to exercise choice of method in attacking practical problems. This is one of the advantages in taking the mathematics as an organic whole, thus cultivating greater resourcefulness than can be obtained by the division into separate subjects. It is desirable that the mathematics course should be directed towards an elementary use of Calculus methods, so that those who remain at school to a later stage may be able to use effectively the elements of Infinitesimal Calculus. The subject-matter may include the elements

of *Algebra, Geometry* (including some *Solid Geometry*), *Logarithms, Trigonometry, Co-ordinate Geometry, Mensuration, Mechanics,* and some *Astronomy,* but especially an easy introduction to *Differential* and *Integral Calculus.*

TIME ALLOTTED TO MATHEMATICS. For such a course five periods per week (each of $\frac{3}{4}$ hr.) are not sufficient, and even six periods will scarcely suffice, unless the mathematical teaching is properly correlated with the science teaching and manual instruction. But the traditional methods of development may be considerably shortened with great advantage.

EXAMINATIONS. The real obstacle to such a course is found in the paralysing influence of the external examinations as at present conducted. They impose an atmosphere of narrowness on the many, and this nominally, but not actually, for the sake of the few. In aiming at the development of the instrument apart from its use, their influence, from a school point of view, is pedagogically unsound. The remedy appears to be to recognise to the full, and act upon, the principle—*Examination is for the teaching,—not the teaching for examination.* At the same time I am bound to confess that many teachers hug their chains.

How great is the obstacle of the present external examinations may be illustrated by a single example :

A teacher handles the subject of rectilinear areas in this way—

(i) If two rectangles have equal bases and equal heights, one may be fitted upon the other.

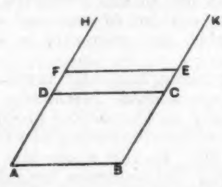
(ii) A parallelogram is equal to a rectangle of the same height on the same base.

(iii) Parallelograms of equal bases and equal heights are therefore equal in area.

(iv) Two duplicate triangles may be fitted together to form a parallelogram. Hence the area of a triangle is half that of a rectangle of the same height on the same base.

Such a procedure runs parallel to the informal treatment in which two duplicate paper triangles are cut out and fitted together, and the whole converted into a rectangle by cutting away and moving a corner piece from one side to the other.

A pupil brought up on these lines takes an external examination in which he is asked to prove that "Parallelograms on the same base and between the same parallels are equal in area." He draws the figure



thus, and says, "Let $ABCD$ and $ABEF$ be two parallelograms on the same base AB and between the same parallels AH and BK." He hesitates to add, "Clearly the parallelograms are not necessarily equal." [No marks ?]

The meeting concluded with the usual votes of thanks to the President, etc.

EUCLID'S DEFINITION OF PARALLEL STRAIGHT LINES CONSIDERED IN REFERENCE TO THE "LINE AT INFINITY."

By J. L. S. Hatton, M.A.

Euclid's definition of two parallel straight lines seems to be the correct one for the absolute ideal of a plane and of a straight line. The conception of a line at infinity, it would seem, should be stated if used in some such words as these. Points may be supposed to exist at such a distance from the points generally considered in a plane that their distances from all such points in the plane may be regarded as equal, *i.e.* the differences of these distances may in general be regarded as negligible. These points, as a first approximation, may, under certain circumstances, be regarded, when considered in relationship to points at a finite distance, as lying on a straight line. Parallel straight lines may be regarded—for most practical purposes—as far as points at a finite distance are concerned, as intersecting at one of these points.

On the other hand, it would seem that the so-called line at infinity is not a straight line nor even a curve. It is a region of the plane or surface considered, which is so distant from the finite region of the plane that, in reference to points in that region, it may be regarded as a straight line or other curve.

The geometrical conception of "the point of infinity on a straight line" may be regarded as based upon the idea that, if A and B are two points at a finite distance apart on a straight line, I is the (or a) point at infinity on the line, if it is on the line and is such that IA may be regarded as equal to IB. If this is the case, a point I' may be taken at a finite distance a from I. Then, if $IA = IB$, it may be assumed that $IA + a = IB + a$. Hence the point at infinity on a straight line is not a single point on the line but an assemblage of points which, on account of their distance and a common property, may for certain purposes be regarded as a single point. The best physical representation of a mathematical point at an infinite distance is a fixed star, say the Polar Star. Yet this is an assemblage of points which is greater than the whole system of points on the earth which it is customary to study.

The geometrical conception of the line at infinity is based on the fact that every straight line at a finite distance in one plane σ, is projected from a point into a line on another plane σ'. In the latter plane σ' there is one straight line v', the vanishing line, which is not projected into a line at a finite distance by the reverse process in the first plane σ. This line is projected into a curve or region in the first plane σ, which, by analogy, is termed the line at infinity in that plane. By analogy it is doubtless convenient to call this locus or portion of space the line at infinity, and for points at a finite distance this region of space has generally the property of a straight line.

There are two ways of reconciling Euclid's definition of parallel straight lines with the conception of the line at infinity.

It is possible to adhere to Euclid's definition of two parallel straight lines in which case it must be assumed that a pair of points can be found one on each of two parallel straight lines situated at such a distance from the finite portion of the plane considered, that with reference to this portion of the plane they may be regarded as one and the same point. In fact, the distance between these two points is infinitesimal in regard to their distance from points in the finite portion of the plane.

On the other hand, the lines considered may be regarded as not parallel but as indefinitely approaching Euclid's definition of parallelism. Such lines may be regarded as intersecting in a point complying with the conception of the point or points at infinity on a straight line.

Looked at from the point of view of projection, it is seen that points on the vanishing line v', or infinitely near it in the plane σ', are projected into points

at an infinite distance in the plane σ. The fact that they are projected to an infinite distance enables those points which are infinitely near in the plane σ' to be projected into points at a finite distance apart in the plane σ. In fact, they form a system of points whose natures and properties are only those of a straight line, because they are so far away. The nature and arrangement of these points depend on quantities and assumptions which, in the case of points at a finite distance, do not require consideration. In fact, by the projection of these points to infinity, small quantities and considerations which could be neglected in the case of points at a finite distance have been changed into finite quantities and considerations of importance.

If parallel straight lines are not regarded as absolutely complying with Euclid's definition of parallelism, but as being straight lines which in the limit approached to Euclid's definition of parallelism—just as we speak of quantities in the calculus which approached the limit zero—and assume that such lines meet at a point at infinity, it is necessary to consider the conditions under which the lines approach to this limit and certain small quantities become important.

Our plane must no longer be regarded as simply a plane. It is something infinitely approaching a plane. Its equation may be written

$$0=k+ax+by+cz+a'x^2+b'^2y^2+c'^2z^2+2gxz+2fyz+2hxy+a''x^3+b''y^3+\dots,$$

where $a'b' \dots a''b'' \dots$ are infinitely small compared with k, a, b, c; in fact, so small that they can be disregarded when points at a finite distance are considered and are only of importance in connexion with points at an infinite distance. These quantities, however, determine the nature of the curve on which the points at infinity may be regarded as being situated.

According to the ordinary conception of the line at infinity, all systems of straight lines which intersect at a point on the line at infinity form parallel systems of straight lines. Since, according to the usual assumption, the line at infinity passes through all these points, it is a line of each of these systems of parallel lines. Therefore all straight lines are parallel to the line at infinity. This paradox holds as long as the line at infinity is regarded as a straight line of the same nature as a straight line at a finite distance. If, however, the line at infinity is regarded as a region of space, as it should be, this paradox disappears. All straight lines at a finite distance, which intersect in a point at infinity, are parallel. Through this point at infinity, different straight lines at infinity can be drawn, but these lines do not necessarily belong to the system of parallel lines, for the essence of parallelism is that the point of intersection, or of approximate intersection, is at an infinite distance from the region of the plane considered. Hence, it follows that the line of infinity has no direction. It is a region of the plane in which lines can be drawn in any direction. There are also points in it which may or may not lie on given straight lines which are entirely at infinity.

Consider the straight line whose equation is

$$\frac{x}{ak}+\frac{y}{bk}=1, \quad \dots\dots\dots\dots\dots\dots (1)$$

and the points whose coordinates are hk and lk. ....(2)

If k be supposed to become infinitely large, the line becomes a line at infinity, and the point a point at infinity.

The general condition that the point (2) should be on the straight line (1) is

$$\frac{hk}{ak}+\frac{lk}{bk}=1$$

or

$$\frac{h}{a}+\frac{l}{b}=1.$$

This is the condition that the particular point at infinity (2) should lie on the particular line at infinity (1). For points at a finite distance, the point (2)

may be regarded as lying on the line (1) when k is infinite, because they are both so distant from the finite portion of the plane.

The equation of a line at infinity should not be written as $0 \cdot x + 0 \cdot y + c = 0$, but as $akx + bky + c = 0$, where k is infinitely small. Thus the equation of a line at infinity is

$$ax + by + \frac{c}{k} = 0.$$

Hence a line at infinity may have any direction according to the ratio of a to b. By giving different values to this ratio, the lines at infinity which are parallel to a given system of parallel lines at a finite distance are obtained.

Our conception of a parabola, which is usually supposed to touch the line at infinity, must be modified. A parabola is a curve which enters and leaves the region at infinity at two points which, with regard to points at a finite distance, may be regarded as coincident; although they may be, and usually are, at a finite distance apart. Hence the inconsistency of supposing that two parabolas whose axes are at right angles touch the same straight line at infinity is done away with. JOHN L. S. HATTON.

Obituary.

CHARLES SAMUEL JACKSON.

CHARLES SAMUEL JACKSON was born at Winchester on the 6th of December, 1867. His father, George Jackson, was a Yorkshireman, and is described as having made his way, without initial advantages, by sheer force of character and great natural ability. His mother is still alive. Charlie, or, to use the "title" rather than "nickname" of later years, "Slide-Rule Jackson," was one of a family of three. It is probable that from both parents he inherited the gifts and attributes which won him distinction, and the affection of all who knew him.

The boy was sent to a Preparatory School at Worthing, and from thence to Uppingham. On the death of George Jackson the family moved to Bedford, Charlie entering the Grammar School in 1881. He rose to be Captain of the School, won a leaving Exhibition, and a Scholarship at Trinity College, Cambridge. He was Eighth Wrangler in 1889, and obtained in the next year a First Class in the Law Tripos.

Of the teachers at Bedford who had exercised the greatest influence upon his intellectual development he used more particularly to refer to W. S. Phillips and E. B. Buck. His frequent letters from Cambridge to the latter were fairly divided between mathematical problems and subtle questions of Law.

For some time he hovered between these two attractions. He read in chambers with Chancellor (now Sir Lewis) Dibdin, K.C., devilling for him during periods of special pressure. But his interest in Mathematics was not allowed to suffer eclipse after he was called to the Bar.

Two Cambridge men write:

"Jackson as an undergraduate was very much the same as we knew him in later life. He thoroughly enjoyed Mathematics and worked hard, but at the same time entered into and appreciated life at college and its opportunities of forming friendships. He and I attended the same lectures in college and were private pupils of Mr. Webb's, of St. John's College, the class consisting of about half-a-dozen people who came out high in their Tripos. One day Webb told us, not altogether seriously, that we were enjoying the May term and not doing enough

work. 'Be industrious, gentlemen,' said he sententiously, 'and you will be happy.' 'But,' replied Jackson, 'you will have very little fun.' Somehow he managed to do plenty of work and have plenty of fun. He formed friendships which lasted all his life, and those who knew him as an undergraduate know what a kindly and joyous spirit has been taken from us." [F. W. D.]

"I recall C. S. Jackson very vividly at Cambridge in 1890. We together attended lectures at Downing by that eminent legal historian, the late F. W. Maitland, and other lectures by Dr. Courtney Kenny. The Maitland lectures on the early history of English law were probably the most brilliant university lectures delivered for many generations at Cambridge. We were all spell-bound, and eagerly entered into the discussions that followed. Jackson was far and away the most striking member of that class, and his industry in becoming acquainted with all historical work and early case-law made a great future for him at the Bar more than probable. On coming down for a time it seemed certain that his great promise at Cambridge would be fulfilled. I saw him occasionally in the Middle Temple Hall on happy festive evenings, and elsewhere, as we were both at the Bar, and his keenness and great knowledge still seemed likely to give him a notable future. But, like Maitland himself, he was constrained to abandon practice, and devoted himself to his life work as a teacher. Later, as we both lived in the same suburb, we talked over the past, its joys and promises, and also over educational problems in which we were both keenly interested. His views were very sound, cautious and wise. I remember well that while he was in favour of the early teaching of calculus he was emphatically opposed to this at an earlier age than fourteen. The C. S. Jackson of 1916 was the C. S. Jackson of 1890. He never aged mentally, and the cheeriness of his outlook, the boyishness of his heart, always remained unaltered. His life was essentially happy, a great deal happier and a great deal more useful to the world than it would have been had he achieved his first ambition and secured fame and fortune at the Bar." [J. E. G. de M.]

Perhaps it was circumstance as much as taste that led to his acceptance of the post of Instructor in Mathematics at the Royal Military Academy, Woolwich, and, much to the advantage of many generations of cadets, he devoted almost five-and-twenty years of his life to his responsible work at that institution. Others, too, have had no reason to regret that the greater part of his energy and ability was given to Mathematics until his death on October 19th, 1916.

All instructors in Mathematics belong to the distributing class. Of the rare few who also belong to the producing class, says Mr. J. P. Kirkman, Jackson was a signal instance. Metaphorically speaking, he was a Builder, an Architect, and an Artist. He was one of those whose career gives support to the theory that given abilities may be made to run in any groove. He might have won distinction in Classics or in Science. He might have been distinguished as a Musician.

He had quite an extraordinary power of concentration: whatever was the business in hand, it was done as if his life depended upon it. Out of everything he touched he extracted all the good—"I will not let thee go, except thou bless me." Of those who knew Jackson well, and whose opinions are worth recording, all are independently unanimous in using the same epithet with respect to him—the epithet *thorough*.

From the first he was a popular instructor, and the delightful caricature of "Slide-Rule Jackson" that appeared a short time before his death in the "Shop" magazine was, with the accompanying letterpress, sufficient evidence that his popularity had long reached a permanent basis. He had a pleasant and easy way of imparting information, and

the ingenuity of his methods was a revelation to the majority of the students in his classes. He was always on the look-out for new methods of presentation, and for new fields in which the principles of mathematics might be applied with effect. To this end he had accumulated an excellent mathematical and scientific library, and with fruitful results.

"Many an officer of the R.E. or R.A. whom one meets out in France," writes a colleague, "still asks for news of 'Slide-Rule Jackson,' and such enquirers always refer appreciatively to his great gifts as a teacher, and to the care and trouble he spent on their training." His modest and tender pride in the performances of the young men who had passed through his hands was infinitely touching to those who knew the depth of feeling of which he was capable. He also spoke with delight of those who responded to the call of duty on the outbreak of the war, appreciating the older and more seasoned material with which for the moment he had to deal, and the surprising results obtainable, under the spur of the intense emotion aroused by the crisis through which the nation was passing, from men whose careers had already opened in other fields. Of all alike, from his earliest pupils, many of whom now are of outstanding rank in the army, to those who had but just left his classes, he was supremely proud. Should these lines by some chance meet the eye of any of those pupils to whom the famous "Ubique" is as the sound of a trumpet call, it will quicken their pulses to learn how much his thoughts were with them in their hour of trial. His voice would swell as he told the tale of some gallant deed. It would break, and his eyes grow dim, as he recalled the lengthening roll of those who had made the great adventure, and who had greeeted "the unseen with a cheer."

When the history of the educational changes of the last twenty years comes to be written, the name of C. S. Jackson will not be forgotten. We need but refer *en passant* to his work for the Association as Member of the Council, as a constant contributor to the *Gazette*, and as Chairman for three years of the London Branch. But, as an old colleague writes, the main work of his life was the breaking down of the barrier that so long existed between Statics and Dynamics, as taught in our schools, and the Applied Mechanics of the engineer. Here he instinctively appealed to his extensive knowledge of the literature of mathematics. The historical development was always of profound interest to him. and he was seldom in fault in assigning the name of the author of a mathematical treatise, or of the discoverer of some out-of-the-way theorem. His mind was stored with a fund of anecdotes, illustrations, and paradoxes, which enlivened the study of the dry bones of the subject, and delighted the budding sappers and gunners. Most of the articles or reviews which he contributed to the pages of the *Gazette* contained instances of these aptly selected historical touches and illustrations.

At Woolwich he was naturally brought into contact with the problems of Gunnery and Military Engineering. This gave the needed impulse to his mind, and he devoted himself *con amore* to the task of breaking down the artificial barrier of which we have spoken. In the early years of the present century it showed signs of giving way. He lived to see its complete demolition. To this desirable and necessary end he contributed in various ways. He preached the new gospel in and out of season. He was able to exert a subtle and wholesome influence by the extensive examining work which he undertook for the Admiralty, Civil Service Commission, and other examining bodies. And by the text-books which he wrote in collaboration with his colleagues at Woolwich, characterised as they were by a delightful freshness of

illustration and fertility of example, he materially hastened the final result. If imitation be the sincerest form of flattery, Jackson and his colleagues were much flattered men.

Jackson died at the post of duty. The war had made holidays impossible for any teacher at the Academy. For nearly two years he had had no rest. One batch of students succeeded another without a break. To some extent the work was new. Lecture-room routine was supplemented by new work for which adequate special preparation was necessary. Surveying, map-reading, and the like were added to the usual duties, so that all the work was, so to speak, on the top of his brain. He responded bravely like the gallant man that he was. He lighted it all up with his quick wit and fertile imagination, but still it was an unwonted strain. All the time there were other preoccupations. The *res angusta domi*, common to so many scientific men in this country, obliged him to add to his exhausting labours a mountain of examination work, and the wonderful thing about it all is that he never let it sink into mere routine. A co-examiner writes: "This side of his work, and his scrupulousness in performing it, have been borne in on me during the recent years in which I have been associated with him. As to his mathematical ability, his invariable courtesy to colleagues, the sparkling wit and humour which often lit up a debate, his generous hospitality and kindness of heart, I have no need to write."

And this brings us from Jackson the official to Jackson the man. His long and intimate association with the *Gazette* is more than sufficient to justify that personal tribute in which we are sure that the troops of his friends, both within and without the bounds of our Association, will long to share. We may well despair of finding adequate terms in which to express the singular fascination of the rare and beautiful character that has vanished from our midst. The warmth of the affection he inspired in so many different types of man and woman throws a vivid light on those elements in his nature which were not long concealed from those with whom he was brought into frequent contact. All recognised an attitude to the world which is variously characterised along the whole gamut from "cheery in temperament" to "animated by a serene optimism." "God's in His heaven. All's right with the world!" may well have been the motto of Jackson, for he was

> "One who never turned his back, but marched breast forward,
> Never doubted clouds would break."

His indomitable industry impressed those who saw the determined resolution with which he faced the future. That future was made doubtful by insecurity of tenure, and this fact necessitated arduous toil outside the duties of his official position. This doubtful future is, unfortunately, shared by almost the whole teaching body of which he was so conspicuous an ornament. But he rarely allowed himself to complain of the harassing conditions under which he fought his life's battle, and with serene and splendid courage he continued to spend and to be spent in the service of his fellows. He combined, alas! in too literal a measure, the two ideals illustrated in the lines:

> "That low man seeks a little thing to do,
> Seeks it and does it.
> This high man, with a great thing to pursue,
> Dies ere he knows it."

Such a life was not "roses, roses, all the way." But if the outward circumstances of life pressed hardly on him, he was amply fortified and sustained by a home life of unalloyed happiness. In 1898 he married Alice Evelyn, the elder daughter of the late Alfred Watmough,

of East Skirbeck, Boston, Lincolnshire. He is survived by her and their nine children. Those who have been privileged to see him in his family life know how entirely happy it was, and what a help Mrs. Jackson was to him in his life and work. No man so happy in his domestic circle as was Jackson could for long look upon the outside world with austere eyes. There was nothing of the pedant in his outlook on humanity. His ideals were as lofty as his impulses were noble. His sympathies were quick, and he could readily adapt himself to the moods of those around him. He enjoyed—none more—the trivial and ludicrous aspects of life; and there was no trace of malice in his keen and ready wit.

His taste in literature was catholic. A few weeks before his death he was discussing with the writer of this notice books appealing to literary and historical instincts so diverse as the *Epistolae Obscurorum Virorum*, Hardy's *Dynasts*, and the satires of Anatole France. In such discussions one was always struck by the breadth and sanity of view, the width of his interests, his dialectical skill, and the transparent sincerity of his convictions.

The richness of his intellectual endowments did not obscure his natural genius for human relationships. The keen edge of his mind was never suffered to conceal his natural sweetness and charm.

Long intimacy and deep personal affection for one, from whom courage and inspiration were being constantly drawn by precept and example, must justify this halting endeavour to convey the salient points in the personality of one who diffused around him something of his own life-long enthusiasm for all that is honest and just, true and pure, lovely and of good report. Suddenly he went to his rest, and has left us older men with heads bowed and with feet still on the road.

Manet Exemplum. Manet Amor. Manet Spes.

MATHEMATICAL NOTES.

506. [C. 2. a. j.] *Squaring the hyperbola and bomb dropping.*

The hyperbola on p. 333, *Math. Gazette*, Dec. 1916, may be considered the equivalent of the figure in the *Principia*, 1713, p. 228, Lib. II, Prop. VIII, employed by Newton for the vertical motion of a body like a bomb, let drop through the air under gravity g, against a resistance growing as the square of the velocity v.

Newton takes the velocity v proportional to AV, OP crossing AR in V, on NQ; and then

$$(1) \qquad AV = OA \tan \omega = OA \operatorname{th} u = OA \sin \phi = MQ, \quad v = H \sin \phi,$$

where H denotes the *terminal* velocity, at which the upward resistance of the air balances the weight of the bomb.

The downward acceleration f at a velocity v is given by

$$(2) \qquad f = g\left(1 - \frac{v^2}{H^2}\right) = g \cos^2 \phi,$$

and the increment of velocity dv is acquired in the moment dt, where

$$(3) \qquad dv = H \frac{Qq \cos \phi}{OA} = H \frac{Qs}{OA} \cos^2 \phi;$$

$$(4) \qquad du = \frac{\text{twice sector area } OPp}{OA^2} = \frac{OY \, . \, Pp}{OA^2} = \frac{OM \, . \, Pv}{OA^2} = \frac{Pv}{OT} = \frac{Rr}{OR} = \frac{Qs}{OQ};$$

$$(5) \qquad dt = \frac{dv}{f} = \frac{H}{g}\frac{Qs}{OA} = G\,du, \quad t = Gu,$$

where G denotes the time of free unresisted fall to acquire the terminal velocity H; thus the time t is proportional to the area of the hyperbolic sector AOP, as proved by Newton.

Further, if dz denotes the increment of the fall z in the moment dt,

$$(6) \qquad dz = v\,dt = HG \tan\omega\,du = 2F\frac{Pv\tan\omega}{OT},$$

where $F = \frac{1}{2}HG = \frac{H^2}{2g}$ is the free vertical unresisted fall for G seconds with average velocity $\frac{1}{2}H$; and $Pv\tan\omega = vp = Tt$, so that

$$(7) \qquad dz = 2F\frac{Tt}{OT}, \quad z = 2F\log\frac{OT}{OA} = 2F\log\sec\phi = 2F\log\text{ch}\,\frac{t}{G},$$

by the previous definition of the logarithm.

Thus for a bomb with $H = 800$, $g = 32$, $G = 25$, $F = 10{,}000$; and let fall from 10,000 feet, $t = 27$, $v = 720$.

If the bomb is projected vertically downward with velocity H, it will continue to move with this constant velocity; and projected with greater velocity, the velocity will diminish down to H ultimately; and the motion may be considered as ensuing from a previous infinite velocity of downward projection.

Produce OP to meet the downward vertical through B in W; here we take the velocity proportional to BW,

$$(8) \qquad \frac{v}{H} = \frac{BW}{OA} = \frac{OA}{AV} = \frac{OA}{MQ}, \quad \frac{f}{g} = 1 - \frac{OA^2}{MQ^2} = -\frac{OM^2}{MQ^2} = -\frac{OA^2}{AR^2} = -\cot^2\phi;$$

and if W rises to w in the moment dt, while the velocity changes dv,

$$(9) \qquad \frac{dv}{H} = -\frac{Ww}{OA} = -\frac{\text{twice triangle } OWw}{OA^2} = -\frac{OW^2}{OP^2}\,\frac{\text{twice sector } OPp}{OA^2}$$

$$= -\frac{OB^2}{TP^2}du = -\frac{OA^2}{AR^2}du = -\cot^2\phi\,du,$$

$$(10) \qquad dt = \frac{dv}{f} = \frac{H}{g}du, \quad t = Gu,$$

as before; and

$$(11) \qquad dz = v\,dt = HG\cot\omega\,du = 2F\frac{OT}{TP}\frac{Pv}{OT} = 2F\frac{Pv}{TP},$$

$$(12) \qquad z = 2F\log\frac{TP}{OA} = 2F\log\tan\phi = 2F\log\text{sh}\,\frac{t}{G},$$

measuring the time backward and forward from the instant where $\phi = \frac{1}{4}\pi$.

In the ascending motion of the bomb, projected vertically upward against gravity and air resistance, we take

$$(13) \qquad v = H\tan\omega, \quad \frac{v}{H} = \frac{AV}{AO}, \quad f = -g\left(1 + \frac{v^2}{H^2}\right) = -g\left(1 + \frac{AV^2}{AO^2}\right) = -g\frac{OV^2}{AO^2};$$

and if V advances to x in the moment dt,

$$(14) \qquad \frac{dv}{H} = \frac{Vx}{AO} = \frac{\text{twice area of triangle } OVx}{AO^2} = \frac{OV^2}{AO^2}d\omega,$$

$$(15) \qquad dt = \frac{dv}{f} = -G\,d\omega, \quad t = -G\omega;$$

and if Vx' is the perpendicular on Ox,

(16) $$dz = HG \tan\omega \, d\omega = 2F \tan\omega \frac{Vx'}{OV} = 2F\frac{x'x}{OV},$$

(17) $$z = 2F \log\frac{OV}{OA} = 2F \log\sec\omega = 2F\log\sec\frac{t}{G},$$

and reckoning the time t backward for the culminating point, it is proportional to the circular sector AOQ, as in the *Principia*.

Projected vertically upward with infinite velocity, the time to the highest point of culmination is $\frac{1}{2}\pi G$, but the height ascended is infinite.

Newton's use of the three letters F, G, H should be followed as the standard notation by all writers on this theory of bomb dropping.

But the Newtonian geometrical method followed above is replaced to-day by the more concise analytical procedure of the Calculus.

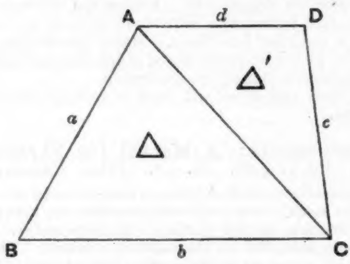
The same theory will apply to a number of associated questions, such as the starting and stopping of a train or steamer, the action of the hydraulic buffer, the rise of a bubble through water, the subsidence of sediment or precipitate, the ascent of smoke up a chimney and the settlement down again in the air; as well as for the original application to Newton's experiments with glass globes, let fall in the dome of St. Paul's.

Staple Inn, Dec. 20, 1916. G. GREENHILL.

507. [K^1. 8. a.] To prove that the area S of a quadrilateral whose sides taken in order are a, b, c, d is given by

$$S^2 = (s-a)(s-b)(s-c)(s-d) - abcd\cos^2\alpha,$$

where $2s = a+b+c+d$, and 2α is the sum of two opposite angles.



$$S = \Delta + \Delta',$$

$$\Delta\cot B = \tfrac{1}{4}(a^2+b^2-AC^2),$$

$$\Delta'\cot D = \tfrac{1}{4}(c^2+d^2-AC^2);$$

$$\therefore \left\{\frac{a^2+b^2-c^2-d^2}{4}\right\} = (\Delta\cot B - \Delta'\cot D)^2$$

$$= \Delta^2\operatorname{cosec}^2 B - \Delta^2 - 2\Delta\Delta' - 2\Delta\Delta'(\cot B\cot D - 1) + \Delta'^2\operatorname{cosec}^2 D^2 - \Delta$$

$$= \tfrac{1}{4}a^2b^2 + \tfrac{1}{4}c^2d^2 - \tfrac{1}{2}abcd\cos 2\alpha - (\Delta+\Delta')^2$$

$$= \ldots\ldots\ldots\ldots\ldots + \tfrac{1}{2}abcd - abcd\cos^2\alpha - (\Delta+\Delta')^2.$$

$$(\Delta+\Delta')^2 = \tfrac{1}{16}\{4(ab+cd)^2 - (a^2+b^2-c^2-d^2)^2\} - abcd\cos^2\alpha$$

$$= \tfrac{1}{16}\{(a+b)^2-(c-d)^2\}\{(c+d)^2-(a-b)^2\} - abcd\cos^2\alpha$$

$$= (s-a)(s-b)(s-c)(s-d) - abcd\cos^2\alpha.$$

R. F. DAVIS.

REVIEWS.

Combinatory Analysis. Vol. II. By Major P. A. MacMahon. Pp. xix+340. 18s. net. 1916. (Camb. Univ. Press.)

It ought to be enough to say of this volume that it is as good as its predecessor; but, since readers of the *Gazette* may like to know what fare it provides, an abstract of the contents may be acceptable. There are five main sections dealing with (1) partitions associated with graphs and generating functions such as Euler's, and those which come from identities such as occur in elliptic function q-formulae; (2) partitions connected with Diophantine inequalities; (3) partitions in two dimensions, with the corresponding theory of lattices; (4) further developments, including solid lattices; (5) symmetric functions of several sets of symbols, distributions, and differential operators. Incidentally we have in (2) a discussion of magic squares, and in (5) the solution of the problem of the Latin square. It may also be noted that on pp. 78-83 we have an analysis, due to Gauss (*Werke*, iii. pp. 461-4), which is fundamental in the theory of theta-functions, and by far the best and most rigorous introduction to it. All Euler's results come from this as special cases; and the Ramanujan (Rogers) formulae (pp. 33-48) may be brought into the same connexion.

As machines, Major MacMahon's differential operators are very powerful, but, like all machines, they have to be directed by a skilled workman, and do not produce anything essentially new. In all this theory we want more and more of genuine arithmetical methods, deduced in whatever way we like from other theories; for instance, theta-function identities give us any number of interesting special theorems in arithmetic, each of which is a problem from the arithmetical side. The classical example is Jacobi's work on the number of representations of a given number as the sum of four squares; he obtained the formula from theta-function theory, and afterwards proved it by strictly arithmetical arguments. Kronecker did much the same thing with bilinear forms, and formulae about them derived in the first place from the theory of elliptic modular functions. Major MacMahon has done his full share in this direction by his frequent use of graphical methods, the importance of which for arithmetic can hardly be overrated.

It is to be hoped that this work will lead to further discoveries by some of our younger analysts. G. B. Mathews.

Theory of Measurements: A Manual for Physics Students. By James S. Stevens. Pp. vii+81. 6s. net. 1915. (Messrs. Constable.)

This book is designed to be used either as a text-book or as a laboratory guide. As a text-book it can hardly be considered satisfactory, for it gives the impression of being a collection of the author's lecture notes, without sufficient expansion to make it intelligible to the solitary student. Thus many students would stumble at the outset over the application of the binomial theorem to questions of probability, where the explanation is limited to two or three lines of the text. The treatment is somewhat unequal: thus in Chapter VI. we find an elementary account of the plotting of the results of observation, while in Chapter II. we have a graph of the Probability curve with an inadequate explanation. In spite of its defects, the book contains much that is stimulating and suggestive to the teacher, each chapter being illustrated by a number of problems or exercises. If a second edition be called for, the usefulness of the book might be increased greatly by a careful revision and the addition of another twenty pages of explanatory matter. H. S. A.

A Foundational Study in the Pedagogy of Arithmetic. By Henry Budd Howell, Ph.D. 1914. (New York: The Macmillan Company.)

The reviewer owes both to Dr. Howell and to the readers of the *Gazette* an apology for his tardiness in introducing them to one another. The apology is all the more called for because Dr. Howell's book belongs to a class with which teachers in this country are insufficiently familiar. Not long ago an

enthusiastic teacher of mathematics complained to a group of friends that his headmaster showed an incomprehensible but persistent reluctance to supply him with coloured chalk. A witty member of the company at once suggested that the headmaster suspected the assistant of an intention to use psychology in his teaching, and had determined to resist the beginnings of evil! An attempt to insinuate the present book into the library of a school where the attitude indicated by this explanation prevails will certainly fail unless very astutely conducted, for there is nothing here but psychology from cover to cover. The substantive contents are divided into two parts. Part I. consists of a review of the more important studies of other authors in the psychology and pedagogy of arithmetic, Part II. is a full report of two experimental investigations by Dr. Howell himself. In Part I., under the title "Genetic Studies," we have a brief account of what is known about the origin and development of the number-sense in primitive peoples and normal children, followed by an interesting summary of the performances of the more famous "lightning calculators." The most significant thing about the power of the calculating prodigy is that it seems generally to grow out of an interest in numbers as such, acquired before he has any knowledge of the use of figures. An analysis of the psychological processes by which these calculators reach their results leads naturally to a section upon the "number forms," first investigated by Galton. Here it is asserted that number forms based upon images of such objects as a clock-face are not retained by adults. It may be worth while to record that several instances observed by the reviewer among post-graduate students contradict this view.

The chapter on "Psychological Studies" describes the results of a large number of researches, some well-known to psychologists, others relatively novel, upon problems connected with the perception of number and the fundamental processes of arithmetic. Many of these are of definite practical importance on account of their bearing upon the relative values of rival methods of computation. In confirmation of the view expressed on p. 83, it is interesting to note Dr. P. B. Ballard's contention that London school children, taught to subtract by the method of "equal additions," show an unmistakable superiority over others in respect both of accuracy and of rapidity.

Under the heading "Statistical Studies" are grouped a number of loosely connected inquiries, of which the most important deal with the distribution of "mental types" among young arithmeticians, the degree of correlation between the different factors in mathematical efficiency, and the much debated question of transfer of acquired ability from one of these factors to others. Most of the statistical evidence on the third point is quoted from Mr. W. H. Winch, inspector of schools under the London County Council; it is to be regretted that Dr. Howell did not turn to another English source in connexion with the second. The investigations of Dr. William Brown, of King's College, London, are by far the most careful and elaborate in this field; they are mentioned in the bibliography, but not referred to in the text.

The chapter on "Didactical Studies" is devoted almost entirely to the many researches that have been made, from the time of Pestalozzi downwards, into the best methods of giving children their first systematic knowledge of numbers and number-combinations. Some of these are described at a length that must be admitted to be disproportioned to their importance. The accounts given of them are, however, to be regarded as introductory to the report of Dr. Howell's first series of experiments which follows immediately. The aim of these experiments was to test, under class-room conditions, the value of Lay's "quadratic pictures" as material for developing children's apprehension of numbers. The results were very favourable to the pictures, and brought out incidentally some noteworthy points with regard to the order in which the cardinal numbers come to full consciousness. It would be interesting to have the results of a careful comparison between the efficiency of Lay's diagrams and those given by Miss Punnett in her *Groundwork of Arithmetic*.

The second series of Dr. Howell's experiments consisted in an application of "Courtis's tests" to the diagnosis of the arithmetical proficiency of a school of 300 children. Mr. Courtis's tests are well-known in the United States.

Their object is to furnish "norms" of efficiency in arithmetic by which a teacher may judge the standard attained by his own pupils at a given age. The tests of accuracy recently circulated by Mr. Siddons may be regarded as having, in part, the same purpose. Materials for "norms" of the Courtis type, adjusted to the conditions in London elementary schools, have recently been accumulated with much industry by Dr. Ballard and published in the *Journal of Experimental Pedagogy*. At the Newcastle meeting of the British Association a further advance towards a satisfactory system of norms for English schools was made in a paper by Prof. Green and Mr. Cyril Burt. Teachers in this country will probably find it best to take their tests from indigenous sources, but they will, nevertheless, discover much interesting and useful matter in the account Dr. Howell gives of the results of his work with the Courtis norms. T. P. N.

Elements of Analytic Geometry. By A. Ziwet and L. A. Hopkins. Pp. vii+280. 7s. net. 1916. (Macmillan Co.)

From the fact that the sections devoted to solid geometry amount to almost half of the pages given to that of the plane, and that yet another twenty pages are absorbed by sections on algebraic and special curves, with empirical formulae, it will be surmised that much material has been dispensed with. This is the case. The text is confined to the most essential properties of the curves. In the fifth chapter, after the circle, we find the first and second derivatives applied and calculated for polynomials. Opinions may differ, but some will feel that this method might as well have been used for the circle, and that the parameter equations for the circle need not have been postponed to the chapter on the parabola. The chapter on the parabola concludes with useful sections on areas and Simpson's Rule. The paragraphs on conics as sections of a cone are good. In the chapters on solid geometry "the idea of the vector is given some prominence, in view of the application to mechanics; and the representation of a function of two variables by contour lines as well as by a surface in space is explained and illustrated by practical examples." The exposition is clear, and the book is well produced. The student who masters the text and works through a careful selection of the examples will be in possession of a considerable knowledge of the value of the tool he is using. The authors had previously published a volume entitled *Analytic Geometry and Principles of Algebra*. The present volume is that part of its predecessor which deals with analytic geometry, which will account to some extent for the early introduction of examples solved by determinants, etc.

Engineering Applications of Higher Mathematics. By V. Karapetoff. Part II. Problems on Hydraulics. Pp. vi+103. Part III. Problems on Thermodynamics. Pp. vi+113; Part IV. Problems on Mechanics of Materials. Pp. iv+81; Part V. Problems on Electrical Engineering. Pp. vi+66. Each 75 c. 1916. (Messrs. Wiley.)

We have not received Part I. of this set of volumes, which treats of problems in Machine Design. We understand that the series is largely intended as a practical protest against the current custom of dropping the mathematical training of the engineer at too early a stage, just when he is beginning "to understand what it is all about." The volumes were conceived under the influence of the "Perry" revolt of two decades ago, and from that point of view they form a systematic exposition of the various ways in which the mathematics already partially or wholly at the disposal of the student may be applied in the various fields of engineering. "The book may be called a summary of the most common engineering applications of higher mathematics, or a mathematical cross-index to engineering text-books." They are extremely well got up and printed; the text is clearly written, and a feature is the number of examples which are worked out at length, often in more than one way. Teachers on this side, who have to do with half-trained students, will do well to consider how far the author's methods, etc., are applicable to their own individual cases. The series is attractively planned, and is not likely to give the pupil a distaste for the efforts which he is summoned to make to escape, for a short time at any rate, from the uninspiring realm of the "rule of thumb."

Dynamics. Part I. By R. C. Fawdry. Pp. viii+177+ix. 3s. 1916. (Bell & Sons.)

This is a companion volume to Mr. Fawdry's *Statics*, Part I., and it is intended for the non-specialist in mathematics. "An attempt has been made to avoid giving the impression that acceleration is necessarily uniform, by introducing examples to be solved by graphical methods and by giving no encouragement to the use of the formulae for uniformly accelerated motion." This procedure no doubt is correct and wise, and many teachers who have had little to do with the teaching of mathematics as part of general culture will be grateful for this and similar hints which may easily be "spotted" throughout the book. That the poundal should go and the dyne be retained is likely to arouse debate in some common rooms, but here again experience is a factor that must not be disregarded. Until we have Part II. we cannot be sure how far the author proposes to limit his application of dynamical principles. As far as he does go, the little book seems to be excellent. The price is perhaps a little too high for many types of secondary schools, but, on the other hand, it is well printed in good type on good paper, with plenty of clear diagrams. It is provided with short historical notes, and as soon as every school has its little library, containing volumes of mathematical biography, Mr. Fawdry, no doubt, will add a slip with hints for those boys who will care to know more about the giants of the past.

Rural Arithmetic. By A. G. Ruston. Pp. xii+431. 3s. 6d. 1916. (University Tutorial Press.)

This book must be described as unique, as may be seen from the titles of some of the chapters—Measurements in the Fields; Water Supply; Brickwork and Building Construction; Dairying; Food Stuffs; Live Stock; Crops; Manures; Soils; Business Letters; Household Accounts. The author's special qualifications for writing an arithmetic of this novel type are set forth in the preface. "Fourteen years' experience as mathematical and commercial master in secondary schools followed by ten years' work as Science Tutor in the Agricultural Department of Leeds University, is probably a unique record." There are slips of minor importance which will no doubt disappear when the inevitable second edition is reached, for there is no work of precisely this type upon the market, and the farmer of the future will have to be a considerable improvement on the farmer of the present. The days of rule of thumb are gone beyond recall, except at the price of ruin in the face of as formidable a competition as the world has yet seen. The multitude and variety of examples based upon original investigations, and the records of sixteen years' work at the University Farm, incidentally convey an enormous amount of practical information to the student, and dispose the budding farmer to think upon lines which have been tested by experience of undoubted value. As an arithmetic pure and simple it seems up to date.

Second-Year Mathematics for Secondary Schools. By E. R. Breslich. Pp. xx+348. 4s. net. 1916. (University of Chicago Press and the Cambridge University Press.)

This is a second edition of a volume published by other authors in 1910, but is practically a new book. The first-year volume was reviewed in our columns in May, 1916, No. 123, pp. 278-9. The main points appear to be: concurrent progress in algebra, plane and solid geometry, and trigonometry; the awakening of the historical instinct by not too frequent but sufficient provision of historical notes, with portraits and short biographies; a definite reference wherever possible to the utility of the subject in its applications at every stage, on the ground that the importance of geometrical facts in everyday life makes a stronger appeal to the youthful mind than any other stimulus the teacher can provide; "explicit exhibits and formulated tests of sound and unsound reasoning"; and the inclusion at this early stage of a considerable body of solid geometry. Those who examine the book will find at the beginning a list of the theorems and axioms already dealt with in the first-year course, and the only knowledge of algebra required is that which is usually acquired after one year's work. Whether the average British boy, with all the present

claims made on his time by an ever-widening curriculum, could really make the progress implied in this course may well be a matter for doubt. It might be done with some ease where it is possible to arrange the time-table so as to admit of intensive work, as, for instance, in the old days when a boy might spend for a couple of years alternate months at Greek, Latin, Greek, Latin, and so on, until he sat for his scholarship at the University. But for boys of the age for whom the book is apparently intended the propriety of intensive work is a matter for psychological investigation, and many old-fashioned people will wag their heads at what the average boy is expected to do in the time. A good deal will depend on the teacher, for we cannot help feeling that mere utility by itself cannot be the whole or the best incentive with which it is possible to supply a boy. The subject-matter is presented as a rule in attractive form, but occasionally statements are made which do not seem entirely necessary, and in a form which will not carry an immediate conviction, for instance: "the differences obtained by subtracting unequals from equals are unequal in the order opposite to that of the subtrahend." Such formalism is repulsive, and we are not sure that at this stage it is necessary to state in general terms the inference from such a sequence as $12=13$, $8>2$, $\therefore 4<10$; but it is evident that a great deal of thought has been spent on the general and the detailed presentation of the group of subjects, and the volume may be safely commended to the attention of teachers as an earnest attempt to secure in a reasonable time a reasonable advance in the elementary branches of mathematics. The writer of this notice has wondered more than once why so careful a writer as De Morgan should have given the date of Fermat's birth as 1595 instead of 1601. The portrait in this volume affords what is probably a clue to the uncertainty, for the date upon it appears to be given as "né en 159. . ." It is not so easy to see why April 23rd is De Morgan's date for the birth of Gauss. On p. 183, lines 2 and 4 up, for Fink, read Finck (cf. l. 8 up, p. 138), and *Geometriae*, l. 3 up, for *Geometria*.

Commercial Arithmetic and Accounts. By Risdon Palmer and J. Stephenson. Parts I. and II. Each 2s. net, or with Answers, 2s. 6d. net. 1916. (Bell & Sons.)

We need not say much more of these two volumes than that they are in their way unique. Modern types of commercial documents are reproduced in colour; every possible stress is laid on fundamental principles; the immense variety of the examples is such that the student will eventually be equipped with an unusual amount of general information not only of commercial matters, but of geography and the social and economic conditions of the world. We look forward with interest to the completion of the book, which comes at a time when our schools will be called upon to provide candidates for posts requiring more than well-drilled machines, and when a distinct cultural value in all commercial training is a matter of the utmost importance in the business community of the future.

Annuaire pour l'an 1917. Publié par le Bureau des Longitudes. Avec des Notices Scientifiques. Pp. 452; +A. 20; +B. 91; +C. 44; +D. 18; +E. 36. 2 frcs. 1917. (Gauthier-Villars.)

The current volume of what we have often called the cheapest book published, and which is in all probability entitled to that claim in spite of a rise of 50 c. in its price, has made its appearance with its usual unfailing punctuality. The Tables, which with those in the volume for 1916 comprise the complete set, relate to: Geography, Statistics, Metrology, Money, Assurance and Compound Interest, and Meteorology. Unfortunately the war has prevented some of the tables from being as fully developed as they will be in the near future. The astronomical part contains tables relative to the deviation from the vertical in France, the variations of weight in different places, and the calculation of altitudes by means of barometrical observations. It also contains data relative to the constellations, stellar parallaxes, double stars, proper motions of stars, and stellar spectroscopy. Many of these have been reviewed in the light of recent records and researches. The "Notices" consist of a most interesting paper by M. Bigourdan on the Babylonian Calen-

dar; a memoir by M. Renaud on the "daylight saving" change in the summer of last year; a valuable note by M. M. Hamuy on the determination of the metre in terms of luminous wave lengths; and a sketch of the life and works of Ph. Hatt, the great hydrographer, with portrait, by M. J. Renaud. There is also a portrait of Commandant Guyou, intended to accompany Mr. Picard's notice of that distinguished scientist in the volume of the *Annuaire* for 1916. We may remind our younger readers that the *Annuaire* has appeared interruptedly since 1796. From 1900 onwards all dates and hours are expressed in civil mean time, from 0^h to 24^h, starting from midnight. From 1912 the hours of the different phenomena here tabulated are expressed in legal time—the mean time at Paris diminished by nine minutes twenty-one seconds.

Revision Papers in Arithmetic, for the Oxford and Cambridge Local Examinations, County Council Examinations for Scholarships, etc. By C. Pendlebury. Pp. xvi + 68 + xviii. 1s. 1916. (Messrs. Bell.)

Of these 200 short papers, to be used as "time-tests," the earlier may be worked mentally, and the rest are intended for written home-work. The very complete table of contents shows the rate of advance. Questions on contracted methods and on elementary mensuration are included. The papers are most carefully graded, with ample revision. The answers to the questions are detachable when necessary.

Sur les Problèmes célèbres de la Géométrie Élémentaire non resolubles avec la Règle et le Compas. By F. Gomes Teixeira. Pp. 132. 1915. (Coimbra, University Press.)

This sumptuous monograph discusses each problem in turn in general terms, followed up in each case with the more famous "solutions." For instance, we find seventeen sections giving typical attacks on the duplication of the cube, dating from Plato and Archytas to Clairaut (1726) and Montucci (1869). The last chapter deals with the impossibility of the solution of the famous problems by rule and compass alone. The contents of this chapter are well up to date, and include a neat proof of the general irreducibility of the equation $x^n = b$, due to the author and published in the Spanish *Revista* in 1914. There are one or two obvious misprints, but they do not interfere with the easy flow of Prof. Teixeira's exposition. From the title it will be gathered that the monograph is written in French.

Opere Matematiche di Luigi Cremona. Tomo Terzo. Pp. xxii + 515. Pubblicate sotto gli Auspici della R. Accademia dei Lincei. 30 l. 1917. (Hoepli, Milan.)

The final volume of this beautiful edition of the collected papers opens with a sketch of the life and works of Cremona by E. Bertini. It is interesting at this period of universal war to recall that the peaceful pursuits of the young student were rudely interrupted in 1848 by the call to arms at the opening of the War of Independence. He especially distinguished himself at the heroic defence of Venice, receiving from his superiors commendation as a "model of military and civil valour." At another period of his life he was drawn into the vortex of political strife, and by his general ability, profound culture, and sheer force of genius, made such an impression on his contemporaries that he was made Vice-President of the Senate. For two years, 1897-8, during the illness of the President, he ruled the Senate "with great ability combined with scrupulous moderation and impartiality." The first of the papers in this volume is the *Memoir on Surfaces of the Third Order*, which was awarded part of the Steiner prize in 1866, a prize which he carried from all competitors on a later occasion. This memoir fills 120 pages. The authorised translation by Maximilian Curtze, under the title *Grundzüge einer Allgemeinen Theorie der Oberflächen in synthetischer Behandlung* (Berlin, 1870), about 25 pp., with 15 pp. of reprint from the *Math. Ann.* of a paper *Ueber die Abbildung algebraischer Flächen*, are the only German contributions in the volume. Memoirs on rational transformations between spaces, and the reprint of the third edition of his *Reciprocal Figures in Graphical Statics* (with a corrected form of which we are familiar from the Hudson Beare

translation of *Il Calcolo Grafico* for the Clarendon Press), are the only papers of any length. Commemorative addresses on Clebsch, Chelini, H. J. S. Smith, W. Spottiswoode, and Beltrami, bear testimony to his literary grace and to his warmth of appreciation.

Plane Analytic Geometry, with Introductory Chapters on the Differential Calculus. By MAXIME BÔCHER. Pp. xiv+235. n.p. 1915. (Henry Holt, New York.)

A straightforward piece of work, with much that will interest the British teacher. "Hesse's Normal Form" is a useful name for $x \cos a + y \sin a = p$, where "normal" is an epithet which implies no connection with the p of the equation, as students might infer. It is well to have names that connote something historically or actually suggestive, *e.g.* the "optical property of the foci" of an ellipse. Can anyone tell us the origin of the term "the magical equation" of the tangent to the parabola, in use in the "fifties" of the last century, for instance in Walton's *Collection of Problems* ? It has a French look about it. There are useful sections on the equations resulting from the Boscovich definition of the conics ; on the parabola as limit of ellipse or hyperbola ; on the invariants and their use ; and on the more elementary applications of the calculus. The elucidatory and explanatory notes are to the point, for instance : "there is no such thing as a 'conjugate hyperbola' . . . ," and those of the interpretations in the quest for loci. Curves with no real points are "no locus." The object of the book is to place the student in the possession of a tool that he can use. The actual acquisition of specific geometric knowledge "is of far less importance." It is no news to some of us, but it is none the less gratifying to see it stated, that "the sources of the best problems in analytic geometry are, to a surprisingly large extent, the English text-books of sixty years ago by Salmon, Puckle and Todhunter." We have heard it claimed that there is "no" problem in the subject, as far as it extended in Salmon's time, which is not to be found hinted at or duly set forth in some form or other in his great treatise. Prof. Bôcher has taken as his motto "one difficulty at a time," and his little volume is deserving of close consideration.

Interpolated Six-Place Tables of the Logarithms of Numbers and the Natural and Logarithmic Trigonometric Functions. By H. W. MARSH. Pp. xii+155. 5s. 6d. net. 1916. (Wiley & Sons ; Chapman, Hall.)

Four-figure tables are useful for teaching purposes and for elementary work in the laboratory, but in industrial and technical problems greater accuracy in the result is of immense importance, and if results are to be exact to the fifth significant figure, a six-place table is necessary. By means of various devices explained in the preface and in pp. ix and x, the reading off is made with great ease and certainty. The tables are : logarithms of numbers (pp. 32) ; logarithmic sines and cosines, tangents and cotangents, interpolated to the second (pp. 93) ; natural sines and cosines, tangents and cotangents (pp. 19) ; length of circular arcs ($r=1$), p. 1 ; and tables of length, area, volume, weight, metric conversion, decimal equivalents, and specific gravity. Blank leaves are left at the end of the book for the reception of further data, or for such additional tables (*e.g.* of e, π, $\sqrt{\pi}$, etc.) as the author has not considered it necessary to include. The figures are large and clear.

Key to Geometry for Schools. Vols. II.-VI. By W. G. BORCHARDT and A. D. PERROTT. Pp. ii+294. 8s. 6d. net. 1915. (Bell & Sons.)

Well printed and got up ; at a price high enough to prevent misuse ; large type, clear figures ; solutions trustworthy and neat.

Arithmetic. Part I. By F. W. DOBBS and H. K. MARSDEN. Pp. xv+353 +xxiv. 1915. (Bell & Sons.)

The great variety and the unusual character of a large number of the examples in this collection of arithmetical exercises will recommend it to the teacher who is glad to get out of the common rut. The ground covered is that required by the ordinary boy, the transition from stage to stage is carefully arranged, and the whole is well calculated to arouse and to maintain a lively interest in the subject. A feature worthy of notice is the prominence given to oral work.

Exercices et Leçons de Mécanique Analytique. By R. de Montessus. Pp. ii + 334. 12 francs. 1915. (Gauthier-Villars.)

We learn from the preface that so much time has to be given to certain fundamental questions, such as the pendulum, the gyroscope, and the like, that candidates for the certificate in Theoretical Mechanics run a serious risk of completing their course without having had brought to their notice a considerable number of principles, theorems, and important applications. Professor de Montessus has gathered together a number of problems such as are likely to throw the principles into relief and to exercise the intuitive faculty: he has arranged them in order of difficulty, with their solutions, and has added other problems, with hints, partial solutions, or without solutions at all. We rejoice to learn that M. Mansion's advice has been placed at the author's disposal. It is something in these days to be assured that he is alive, and we hope that he will be spared to resume his functions in Ghent. Unfortunately we have been unable to discover what has become of his colleague on *Mathesis*—Prof. Neuberg, and have long feared the worst. It is pathetic also to read the closing lines of the preface: "Malgré les circonstances du temps présent, M. Gauthier-Villars a bien voulu imprimer ce livre et accueillir le vœu d'un proscrit."

About eighty pages deal with centres of gravity of curves and surfaces, attractions and potential. In the second part, which is more than twice as long as the first, we find moments of inertia, virtual work, Lagrange's equations, motion of a rigid body, stability, small oscillations of a system about a position of stable equilibrium, and impact. A useful Note of sixty pages on Elliptic Integrals in the real domain gives in small compass the essentials of the theory of elliptic functions in a form more conveniently adapted for calculations than can be readily derived from the usual courses in analysis, in which the main stress is laid on the functional point of view.

Test Questions in Junior Algebra. Edited by F. Rosenberg. Pp. ii + 113. 1s. 3d. with Answers; 1s. without. 1916. (University Tutorial Press.)

This collection of carefully selected exercises is intended as a supplement to the ordinary text-book for pupils working up to the standard of the Junior Locals. The order of the preliminary exercises is that followed in Mr. Cracknell's *Junior Algebra*, and the individual teacher must judge for himself how far his special circumstances require these additional tests.

Theory of Errors and Least Squares. A Text-book for College Students and Research Workers. By L. D. Weld. Pp. xii + 190. 5s. 6d. net. 1916. (Macmillan Co.)

The method of least squares, as De Morgan has said, is the method of finding the most probable truth, when a number of discordant observations has been made upon a phenomenon. But things have moved since his day, and from his concluding remarks that astronomy is perhaps the only science in which so delicate a test is absolutely necessary we have a fair measure of the extent of the application of a method of computation first hinted at by Cotes, habitually used by Gauss from 1795, and now available in every field of investigation in which statistics intervene. This little volume, the basis of which is the lecture notes used by the author for the last twelve years, is intended to be a text-book for undergraduates and a "handy reference which any research worker can read through in an evening or so and then put into immediate practice." We are inclined to think that the author is somewhat optimistic in his view of the time it takes to get a working knowledge of the subject, and the book would be the handier as a work of reference had the table of contents been supplemented by a handy index, the need of which is only partially met by Appendix X., a collection of important definitions, theorems, and of rules and formulae, which, oddly enough, contains no definition of "a law of error." It is to be hoped that this blemish will disappear from a volume which certainly fills, and adequately fills, a gap. It is a clear and happily arranged exposition of the essentials which will prove

comparatively easy reading for the student who has had a mathematical training of the requisite breadth—which after all is limited. Nearly a quarter of the book is given to exercises and illustrative examples from various branches of science.

George Boole's Collected Logical Works. Vol. II. Laws of Thought. Pp. xvi+448. 15s. net. 1916. (Open Court Co.)

Boole and De Morgan alike were severely handled by "the arch syllogist," Sir William Hamilton, for "meddling with logic by help of mathematics." Between 1839 and 1844 the researches of Boole had led him to the conviction that no serious difficulties lay in the way of the construction of a calculus of deductive reasoning. The mathematician became merged for the time in the student of mental science, and in 1847 the publication of De Morgan's *Formal Logic* acted as the necessary catalyser. In rapid succession appeared his *Mathematical Analysis of Logic* and his *Calculus of Logic*, and "genius and patience combined" brought into existence after six years more of steady labour, the famous *Laws of Thought*. The edition of the *Collected Logical Works of Boole*, of which the second volume lies before us, is shortly to be followed by a companion in the *Logical Works of De Morgan*. The errata in the original have been corrected, and with this exception we have in Vol. II. an exact reproduction, title page included, of the *Laws of Thought* as it appeared in 1854. The first volume is not ready yet. It will contain a portrait and bibliography of Boole, with an appreciation of his logical work, its place and influence, from the competent pen of Mr. Jourdain. In the circumstances, it seems better to defer a review of Boole's place in science until Volume I. makes its appearance. In the meantime we need make no excuse for reproducing what De Morgan says of the friend and correspondent of more than twenty years :

"His first paper in the *Cambridge Mathematical Journal* contains remarkable speculations which can here be described only in general terms, as extensions of the power of algebraic language. These papers helped to give that remarkable impulse which algebraic language has received in the interval from that time to the present. . . . That peculiar turn for increasing the power of mathematical language, which is the most characteristic point of Dr. Boole's genius, was shown in a remarkable way in his writings on Logic. Of late years,* the two great branches of exact science, Mathematics and Logic, which had long been completely separated, have found a few common cultivators. Of these Dr. Boole has produced far the most striking results. In alluding to them we do not say that the time is come in which they can even be generally appreciated, far less extensively used. But if the public acknowledgment of progress and of genius be delayed until the whole world feels the results, the last century, which had the lunar method for finding longitude, ought to have sought for the descendants of Apollonius to reward them for his work on the Conic Sections. Dr. Boole's system of logic shows that the symbols of algebra, used only to represent numbers, magnitudes, and their relations, are competent to express all the transformations and deductions which take place in inference, be the subject what it may. What he has added may be likened to a new dictionary, by consultation of which sentences written in the old grammar and syntax of a system take a new and true meaning. No one is ignorant that the common assertion, 'Nothing is both new and true,' is a perfect equivalent of 'Everything is either old or false, or both.' Dr. Boole showed that a school-boy who works a certain transformation, such as occurs in many a simple equation, has the form, though applied to very different matter, of this logical passage from one of two equivalents to the other. Taken alone, this is a pretty conundrum, if any one so please. But when looked at in the system of which it is a part, and when further considered as the produce of a mind which applied the same power of thought with rare success over the whole of the higher Mathematics, those who so look, and so consider, are justified in presenting it as a type of genius, and as a specimen which may give those who are not mathematicians a faint notion of an originality of speculation, which, applied to the progress of science, has attained most useful results, and made a lasting name."

In the *Budget*, he writes shortly after the receipt of the news of the death of the author of the *Laws of Thought* :

"I might legitimately have entered it among my *paradoxes*, or things counter to general opinion ; but it is a paradox which, like that of Copernicus, excited admiration from its first appearance. That the symbolic processes of algebra, invented as tools of numerical calculation, should be competent to express every act of thought, and to furnish the grammar and dictionary of an all-containing system of logic, would not have been believed until it was proved. When Hŏbbes, in the time of the Commonwealth, published his *Computation or Logique*, he had a remote glimpse of some of the points which are placed in the light of day by Mr. Boole. The unity of the forms of thought in all the applications of reason, however remotely separated, will one day be matter of notoriety and common wonder : and Boole's name will be remembered in connexion with one of the most important steps towards the attainment of this knowledge."

* This was written in 1864.

Preliminary Geometry. By F. Rosenberg. Pp. viii+220. 2s. (With or without Answers.) 1916. (University Tutorial Press.)

We are sorry to say that in these days it constitutes a special claim if the compiler of a book can say in his preface : " Pains have been taken to render the text attractive to the eye, by means of wide spacing between successive lines and the avoidance of crowding. The whole of the matter in this book could, at the expense of the sight and temper of the learner, have been comfortably compressed into half the space actually occupied." Fortunately there are many text-books in which these pains are taken as a matter of course—Mr. Fawdry's volume above is a good case in point—and which claim no special credit for the fact, The public cannot be too grateful to the publishers who do make a special study of size of figures and letters, spacing, and all the rest of it. Mr. Rosenberg's interesting venture consists in " a judicious blending of the theoretical and practical sides." Experience has shown that practical work should not be unnecessarily extended, that it should be perfectly definite in its aim, and that the most successful arrangement of the practical exercises is that which leads either to the direct discovery by the student of the geometrical truths conveyed in the important propositions, or to such a wholesale conviction of their plausibility as will shortly lead to the desired end. The only " proof " of such puddings is in the eating, and for that purpose we may recommend the book to teachers as well worthy of careful consideration. For the private student it seems to us an admirable introduction to the subject.

Statics, A First Course. By C. O. Tuckey and W. A. Nayler. Pp. 300. 3s. 6d. 1916. (Clarendon Press.)

Mach has said that the lever and the inclined plane are the only self-created ideal objects of mechanics which completely satisfy the logical demands we make upon them. Every child who has seen others on a see-saw, and who has played with the nursery scales, has an instinctive notion of some part of the principle of the lever. In the infancy of the science " Statike " was " an Art Mathematicall, which demonstrateth the causes of heavinesse and lightnesse of all things " : as we are reminded in *John Dee, his Mathematicall Preface*. And he continues : " forasmuch as, by the Bilanx, or Ballance (as the chief sensible Instrument,) Experience of these demonstrations may be had : we call this Art Statike : that is, the Experiments of the Ballance." So moved indeed is he by the " mercifull goodnesse " of the Creator, who in making His creatures used " three principall ways, namely Number, Weight, and Measure," that he bursts into a panegyric of the " wondrous Wisedome and Goodnesse which may be shown forth (to the weaklings in faith) by means of " this third Art or key. Anticipating the surprise of his reader at this outburst from a heart overcharged, he adds : " Marvell nothing at this pang (godly friend, you gentle and zealous student). Another day, perchance, you will perceive what of occasion moved me." The sternly practical is usually the motive of discovery. It did not need the genius of an Aristotle in the market place to discover that his purchase of a bushel of beans was short by an *xestes*. But to know the fact and to find the why thereof are different things. By the time of the great Stagyrite, ideas of the principles of the balance and the lever had begun to be formulated : men were aware of a something corresponding to what we call the centre of gravity : Archimedes was not in a position to define κέντρον βάρους, which was in his time already a term in general use but of shadowy significance. What Archimedes did do was to take the first steps towards the mathematical proof of a mechanical principle. It may be, as Vailati suggests, that the search for the properties of the centre of gravity led the philosopher of Syracuse to an endeavour to connect that mysterious point with the properties of instruments with which he was interested from the point of view of theory. The substitution of an ideal instrument for a practical, of symmetrically arranged small weights for one large weight, were steps to which Archimedes was led in the *furor demonstrationis* by an unconscious utilisation of previous reflections upon the centre of gravity. And although later writers saw that statical moments and the theory of the centre of gravity were in some way involved in the principle of the lever, it was not by this route that they decided to lead their students in the search for statical principles. There came a time when the

principle of the parallelogram of forces was, so to speak, in the air. In one and the same year, 1687, we have enunciations of the principle from Newton, Varignon, and Lami. For long it has been the tradition of British text-books to take the parallelogram of forces as the basis on which is erected the mechanical edifice. From this tradition Messrs. Tuckey and Nayler have cut themselves adrift. Their "First Course" boldly opens with the law of the lever, from which it is the easiest of steps to moments about a point, and to the notion of the centre of gravity. The parallelogram of forces is hinted at on p. 74, stated on p. 113, and "proved" and discussed on pp. 266→. So marked a departure from the usual course on the part of two chartered libertines should arrest the attention of teachers, and prepare them for yet further surprises. Chapter II. shows how forces are resolved, and the principles of the first two chapters are then applied to the simpler forms of machines. So far, the work has run *pari passu* with experiment. Geometry is now introduced. Graphical methods are carefully explained. The notation of R. H. Bow, who in the early seventies familiarised British readers with the product of the genius of Culmann, is rightly retained. It is strange that such a chapter as that entitled "The connection between the Principles" should have for so long been missing from our elementary text-books. To this we must refer the reader, with the suggestion that he cannot afford to give Messrs. Tuckey and Nayler's little book a merely casual glance. For their constant reference to fundamental principles is, if we are not mistaken, the great *raison d'être* of their little book.

CORRESPONDENCE.

To the Editor of the *Mathematical Gazette*.

Dear Sir,

The death of my friend C. S. Jackson makes it impossible for me to reply in detail to the note signed by him and Mr. A. Lodge on p. 311 of the October *Gazette*. I can reply only generally by saying that I do not admit that the note correctly represents either what I actually wrote (July *Gazette*, p. 296) or the necessary deductions from what I wrote.

The quotation in the first paragraph touches a question on which I should like to make some observations later on.

The last paragraph introduces a new point, which I heartily welcome. If this discussion results in a greater use of arithmetical quantity in elementary teaching, in the place of mere number, it will have done some good. But, even so, I should hope that arithmetical quantities which are shown as the result of multiplications or divisions, and which have to be added or subtracted, would be enclosed in brackets, unless precautions as to spacing, etc., make this unnecessary.

As there are a great many people who object to the rule laid down by teachers of arithmetic with regard to this use of brackets, could not the matter be considered by a small committee? There is more to be said on the subject than has yet been said in the *Gazette*.

W. F. Sheppard.

I have to thank Mr. Sheppard for most courteously sending me the above letter to read before passing it on to the Editor. I much appreciate this tribute of respect to our friend, whose loss we mutually deplore.

Alfred Lodge.

Sir,

The following argument seems to have been overlooked by those who consider it unnecessary to insist on the convention of priority of multiplication and division over addition and subtraction.

The ordinary boy, when learning the rudiments of Algebra, is greatly helped by frequent references to what he has done in Arithmetic. He is encouraged to check his algebraical work by simple numerical sub-

stitutions, both for the sake of accuracy, and for the sake of a general appeal to his commonsense reasoning in the concrete.

One of the most frequent sources of error is the confusion between $a+bc$ and $(a+b)c$, and this is especially noticeable when numerical values are substituted for letters.

A few weeks' practical experience in dealing with this error in boys who are not particularly brilliant will convince the most sceptical mathematician that, however needless the priority convention may be in ordinary Arithmetic, he must insist on it at all times, whether he wishes to or not. He will find it impossible to get on without it.

It is useless to argue that $a+bc$ ought to convey its real meaning to the average boy, when he substitutes numerical values. The boy will write $8{\cdot}2+1{\cdot}8\times 14{\cdot}5$, and unless he has had this apparently unnecessary rule drilled into him, the chances are about even that he will write the result as 145.

It is from the psychological point of view a curious fact that this is especially noticeable, when the numbers are so heavy as to require a conscious effort for the working out of their product.

This type of mistake is continually cropping up in the practical application of mathematics to formulae and to physical problems. Such a mistake as $1{\cdot}7+8{\cdot}3(t-4)=10(t-4)$ is all too common, and we shall not improve matters by deliberately stating that $1{\cdot}7+8{\cdot}3\times 9$ may be taken to mean either $(1{\cdot}7+8{\cdot}3)\times 9$ or $1{\cdot}7+(8{\cdot}3\times 9)$.

Royal Naval College, Osborne. R. NETTELL.

DEAR SIR,

I would have much preferred that others should have replied to Prof. Hill's letter on p. 15 of the present volume of the *Gazette*, for, alas, it cannot now be a joint reply; but as he specially calls on me to controvert, if I can, his argument on p. 281 of the last volume that the application of Rule 1 to such an expression as

$$9-6\div 3\times 2+4$$

is illegitimate because $6\div 3\times 2$ is of doubtful meaning, I feel bound to say a few words.

Putting aside for a moment the meaning to be attached to $6\div 3\times 2$, there is no doubt in my mind, and I believe the great majority of your readers will agree with me, that it is a number which has to be subtracted from the sum of 9 and 4. It constitutes a 'term.' If arithmetical and algebraic conventions are to be as nearly as possible identical, there is no alternative.

Rule 1 accepts the existence of terms, and indeed may be said to define them; terms being those quantities which are separated from each other by + or − signs. That completes my answer to Prof. Hill's specific question.

With regard to the term quoted by Prof. Hill, viz. $6\div 3\times 2$, my own feeling is that it ought not to be written without brackets for the reason I gave before, viz. that, if it means $(6\div 3)\times 2$, which is in accordance with Rule 2, it ought to have been written in the unambiguous form—$6\times 2\div 3$; consequently, if it is written with $\div 3$ in the middle, the inference is almost irresistible that it must be intended for $6\div(3\times 2)$, especially if it is read as "6 *divided by* 3 *times* 2," which is a perfectly fair reading. It is no longer a mere beginner's difficulty; experts also would be in doubt.

But that is no reason for discarding Rule 1: this particular difficulty has nothing whatever to do with that Rule! That is why we called it a red herring. ALFRED LODGE.

Charterhouse, Godalming, 30th January, 1917.

DEAR SIR,

Prof. Hill's question on page 281, vol. viii., seems irrelevant. Taking the example $9-6\div3\times2+4$, it is obvious that the presence of the $9-$ and the $+4$ have nothing whatever to do with the interpretation of $6\div3\times2$, so that Rule 1 may stand. I should vote for the retention of Rules 1 and 2 on the ground that, after using them, boys will be able to see more clearly why brackets become necessary. It is a valuable lesson for them to learn the difference between $3+4\times5$ and $(3+4)\times5$; and I fail to see that any real effort of memory is required.

Rule 3, about "of," is new to me, and is ambiguous; surely $\frac{3}{4}$ of $4+5$ is never interpreted to mean $\frac{3}{4}$ of 9!

One word more about "of." The expressions $\frac{3}{4}$ of a cake, $\frac{3}{4}$ of 20, are soon understood by youngsters; but $20\times\frac{3}{4}$ is a mystery. Yet hundreds of boys are allowed to write

$$\tfrac{3}{4} \text{ of } 20 = 20\times\tfrac{3}{4} = \tfrac{20}{1}\times\tfrac{3}{4} = \text{etc.}$$

This, at the early stage, should be deprecated.—Yours faithfully,

R. W. GENESE.

REPORT OF SYDNEY BRANCH FOR 1916.

THE past year has been a very successful one. The membership now stands at 50, 14 of whom are members of the Mathematical Association. Three meetings have been held during the year, the attendances have been good and the addresses excellent. In November, 1915, F. G. Brown, B.A., B.Sc., formerly Director of Studies, Australian Royal Naval College, gave an address entitled, "Some Real Applications of Elementary Mechanics." In August, 1916, E. M. Wellisch, M.A., Lecturer in Applied Mathematics, University of Sydney, read a paper on "Some Famous Problems in Mathematics and Physics." In September, 1916, D. K. Picken, M.A., Master of Ormond College, University of Melbourne, gave an address entitled "A Modern Equivalent to Euclid," a plea for a modern, thoroughly commonsense statement of geometrical theory with all the merits of Euclidean presentation. One important feature of our work is the circulation of the *Mathematical Gazette* among the members of the Branch.

R. J. MIDDLETON, *Hon. Secretary.*

THE LIBRARY.

CHANGE OF ADDRESS.

THE Library is now at 9 Brunswick Square, W.C., the new premises of the Teachers' Guild.

The Librarian acknowledges, with thanks, the presentation, by the Clarendon Press, of a copy of *Statics: A First Course.* C. O. Tuckey and W. A. Nayler.

SCARCE BACK NUMBERS.

Reserves are kept of A.I.G.T. Reports and Gazettes, and, from time to time, orders come for sets of these. We are now unable to fulfil such orders for want of certain back numbers, which the Librarian will be glad to buy from any member who can spare them, or to exchange other back numbers for them:

Gazette No. 8 (very important).
A.I.G.T. Report No. 11 (very important).
A.I.G.T. Reports, Nos. 10, 12.

GLASGOW: PRINTED AT THE UNIVERSITY PRESS BY ROBERT MACLEHOSE AND CO. LTD.